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Computer Vision Approach for Ultrasound Doppler Angle Estimation

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Doppler ultrasound is an important noninvasive diagnostic tool for cardiovascular diseases. Modern ultrasound imaging systems utilize spectral Doppler techniques for quantitative evaluation of blood flow velocities, and these measurements play a crucial rule in the diagnosis and grading of arterial stenosis. One drawback of Doppler-based blood flow quantification is that the operator has to manually specify the angle between the Doppler ultrasound beam and the vessel orientation, which is called the Doppler angle, in order to calculate flow velocities. In this paper, we will describe a computer vision approach to automate the Doppler angle estimation. Our approach starts with the segmentation of blood vessels in ultrasound color Doppler images. The segmentation step is followed by an estimation technique for the Doppler angle based on a skeleton representation of the segmented vessel. We conducted preliminary clinical experiments to evaluate the agreement between the expert operator's angle specification and the new automated method. Statistical regression analysis showed strong agreement between the manual and automated methods. We hypothesize that the automation of the Doppler angle will enhance the workflow of the ultrasound Doppler exam and achieve more standardized clinical outcome.

KEY WORDS: Automated measurement, automated object detection, biomedical image analysis, clinical application, computer analysis, computer, vision, image analysis, ultrasound, color Doppler, vascular

INTRODUCTION

Before the development of diagnostic ultrasound scanners, angiography was the sole invasive diagnostic tool for assessing vascular disorders. The invasive methods had many drawbacks such as patient discomfort, complications, lack of functional information about the observed lesion, and inability to provide history information of the disease[1]. Due to the need for additional information about the vascular disease, considerable effort has been put in the development of noninvasive diagnostic methods. The earliest noninvasive methods relied on measuring peripheral vessels systolic blood pressure that correlate with disease state. However, blood pressure is only one element of the physiologic assessment. The development of the audible continuous wave (CW) Doppler in the late 1950s enabled noninvasive evaluation of the arterial system. However, CW Doppler was unable to determine the exact site of flow. The pulsed wave Doppler was developed in the early 1970s, which enabled the selective sampling of blood flow at a desired depth. The introduction of duplex ultrasound scanners that employ both anatomical ultrasound imaging and spectral Doppler quantification since 1974 has changed the field of vascular disease diagnosis and treatment. The typical form of a duplex system is the integration of a single-gate Doppler system with the imaging one. In the early 1980s, another important advance occurred by combining real-time color-coded Doppler flow imaging with anatomical imaging and spectral Doppler. The real-time color flow images allowed better interrogation of the underlying vessels by guiding the

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placement of the Doppler gate. Nowadays, surgical decisions are often made solely based on the outcome of the Doppler ultrasound exam, avoiding invasive methods altogether.

Standard Doppler ultrasound techniques measure only the component of the blood flow velocity, which is parallel to the ultrasound beam. Therefore, knowledge of the angle between the flow direction and ultrasound beam (Doppler angle) is needed in order to go from the measured velocity projection component to the actual velocity vector. Modern ultrasound systems offer a graphical interface tool for the operator to specify the Doppler angle for every site of interest and for every interrogated vessel during the diagnostic exam. This repetitive manual process can be time-consuming and inconsistent among all operators, even within the same clinical department.

The accurate measurement of blood flow velocity with Doppler ultrasound plays a crucial rule in the diagnosis of vessel stenosis. Investigators have confirmed that the average Doppler velocity rises in direct proportion to the degree of stenosis as determined with angiography[2,3]. A recent consensus paper by a panel of experts in the field of vascular ultrasonography[4] specified that the peak systolic velocity measurement in Doppler ultrasound should be used along with the grayscale and color Doppler plaque findings to diagnose and grade the internal carotid artery stenosis. Some of the concerns raised by the panel are the lack of standardization of the vascular exam among different laboratories and the errors introduced in the velocity measurements due to the errors in the Doppler gate positioning and angle setting. To get accurate velocity measurements, the Doppler gate has to be positioned at the site of maximum stenosis, and the Doppler angle cursor has to be accurately aligned with the vessel axis. It is recommended to either fix the Doppler angle to be exactly 60° or maintain an angle of 60° or less. Another recommendation made was to develop improved methods for calculating velocity with angle correction to eliminate or minimize the inconsistency in velocity measurements as the Doppler angle of insonation is changed[4].

There have been many clinical Doppler studies that focused on determining how the accuracy of the velocity measurement affects the accuracy of the diagnostic evaluation of arterial stenoses[5,6]. Other publications focused on the interobserver variability effect on the peak velocity measurements[7,8]. Doppler angle is a significant source of measurement error in Doppler velocity measurement. It is well established, via in vitro string phantom studies, that systematic overestimation errors of approximately 18% exist and are increased at higher Doppler angles[9,10]. Steinman et al.[11] showed that overestimation errors mainly result from the dependence of peak velocity measurement on the Doppler angle. Recently, Lui et al.[12] conducted a study to assess the error and variability that results from human factors in Doppler peak velocity measurement. Doppler angle was one of the most significant sources of error and variability. Inaccurate angle increased the variability in measurements from 1% to 2% to 3% to 9% with in vitro flow phantom experiments. Dejong[13] from the Intersocietal Commission for the Accreditation of the Vascular Laboratories (ICAVL) commission reported that as high as 35% of the applications for accreditation received by the ICAVL demonstrate improper angle correction techniques, making angle correction issues one of the most common causes for delayed decisions.

Prior attempts to automate Doppler angle estimation have been primarily initiated by ultrasound manufacturers and are available as patent publications. Lihong et al.[14] applied a local search method based on the grayscale intensity or color Doppler pixels to estimate the vessel edge points, which are used to calculate the vessel slope at the current Doppler gate position. Criton and Routh[15] suggested a new acquisition mode that utilizes two Doppler beams that crossover at the Doppler gate position. The two returning Doppler signals are used to calculate two orthogonal components of the velocity vector from which the true velocity vector can be estimated. So far, the prior techniques have not appeared commercially, and no clinical trials have been published to validate the accuracy and efficacy of the proposed techniques.

Ultrasound systems can greatly benefit from recent advances in the field of computer vision in terms of both the accuracy of the results and speed enhancements of modern image analysis algorithms. In this paper, we present a computer vision system that analyzes raw color Doppler ultrasound images to estimate the optimal Doppler angle. The algorithms were integrated within the signal path of a Philips ultrasound system, the iU22[16], and clinical experiments were conducted to evaluate the accuracy and performance of the automated technique against the traditional manual angle estimation currently used by ultrasound system operators.

METHODS

The first step in our technique involves data acquisition and preprocessing. In order to deal with the blood flow pulsatility within arteries, we capture a number of consecutive raw color Doppler frames that encompass at least one heart cycle. These frames are temporally averaged to remove the effect of pulsatility, resulting in a single temporal-average image, which is then thresholded using the ultrasound system's internal threshold processing used for combining the color Doppler image with the grayscale B-mode image. The resultant binary image is a good representation of the present vessels. However, due to a well-known artifact of the color Doppler imaging called "color bleeding", it often suffers from artificially connected vessels. The color bleeding artifact occurs when the color spills out of the vessel and writes on the vessel walls or surrounding plaque and tissue[17]. This artifact is related to the spatial and temporal resolution differences between color Doppler and B-mode imaging and the need to overlay these two modes in a single image. When color bleeding occurs, adjacent vessels appear as if they are connected, as illustrated in Figure 1. Ultrasound clinicians can minimize the color bleeding effect by optimizing color gain and write priority. However, the artifact may not be completely avoidable due to incomplete vessel boundaries, which occur due to imperfect insonation angles. In addition, subtle bleeding may not be noticeable by the human visual system, but it will fail the angle automation due to distorted vessel morphology.

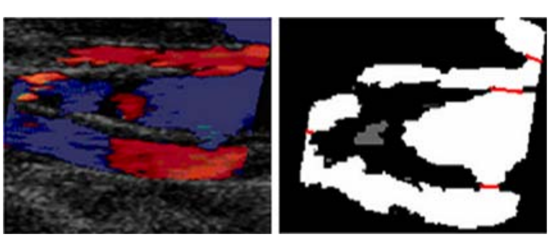

Fig 1. ***Left*** **A color Doppler image of peripheral vessels. The color bleeding artifact artificially connects the vessels together.** ***Right*** **A binary representation of the color Doppler image.**

The second step in our technique applies a vessel segmentation procedure in order to disconnect vessels, which have been artificially linked due to color bleeding. Our approach relies on a shape decomposition technique that was designed to decompose natural objects into meaningful parts that agree with the human visual system partitioning[18]. Shape is an important visual feature used for object recognition, image database retrieval, image matching, and image analysis. There are many shape description and representation techniques in the literature[19]. Structural shape description is one of the major shape description methods, where a shape is described in terms of simpler shape parts and the relationships between these parts. Shape decomposition techniques can be classified into region-based versus boundary-based partitioning. Examples of region-based decomposition are overlapping disks, maximally convex parts, generalized cylinders, and superquadrics. Examples of boundary-based decomposition are high-curvature points, constant-curvature segments, and polygonal approximations. Many theories also exist in the literature to describe the correct and intuitive partitioning scheme for shapes based on psychophysical and ecological evidence [18]. In our case, the problem of linked, elongated, convex vessels is modeled as a partitioning problem. The segmentation algorithm starts by detecting the negative curvature minima of a complex object consisting of many linked vessels. Then, a hierarchical partitioning scheme is applied to the complex binary object to disconnect linked objects.

Figure 2 shows the steps of the hierarchical decomposition of the connected vessels of Figure 1. Figure 2a illustrates the complex binary object of Figure 1, with four connected vessels along with the detected negative curvature minima. A partitioning line that involves negative curvature minima is chosen by the algorithm to split the complex object into two simpler objects, as shown in Figure 2b. Figure 2c and d illustrate another step of the hierarchical decomposition technique where another partitioning line is chosen to split one of the two objects of Figure 2b into two simpler objects. Figure 2e illustrates the resulting parts after two partitioning steps. This partitioning process continues iteratively until all negative curvature minima are consumed. Figure 2f shows the final result of the partitioning process with the successful segmentation of the four vessels. More details about the segmentation technique can be found in[20].

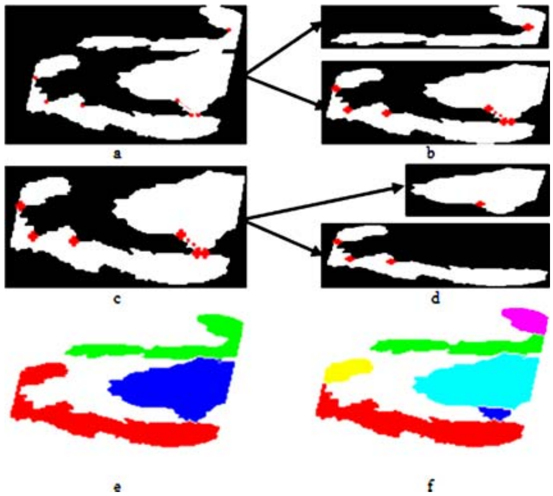

Fig 2. A hierarchical vessel segmentation approach. a A complex binary object of four connected vessels. b The first partitioning stage output with two simpler parts. c Another complex object of two connected vessels. d The second partitioning stage output with two simpler parts. e An intermediate stage of the vessel partitioning scheme. f The final partitioning with four distinct vessels.

The third step is to find a good representation of the segmented vessel that will allow efficient and robust estimation of the Doppler angle in any arbitrary site within the vessel. In the computer vision field, there are many shape representation methods; some are contour-based, such as chain codes and B-splines; some are region-based, such as convex hulls and skeletons[19]. We chose the skeleton representation as an efficient and robust representation of the segmented vessels. Maillet and Sharaiha[21] dedicated a chapter in their book to surveying the informal and mathematical definitions and algorithms that deal with object skeletonization. Informally, a skeleton is a thin central structure that uniquely represents the object. Formally, a perfect skeleton should be totally contained in the object, be one pixel wide, preserve the object topology, and allow reconstruction of the object. However, some of these conditions may be relaxed depending on the application. One early mathematical model of a skeleton was introduced by Blum[22] as the locus of centers of maximal discs totally contained in the object. Another model was introduced by Montanari[23] using wave propagation with constant speed from each border point towards the inside of the object perpendicular to the border; the wavefront intersection points belong to the object skeleton. We applied a recent skeletonization method based on a graph-theoretic approach, introduced by Bitter and Kauffman[24], to detect vessel skeletons. This graph-theoretic approach maps the object pixels into graph vertices and the pixel neighbor relations to graph edges. The technique integrates a modified version of Dijkstra's shortest path algorithm with the inverse of the distance map of the object (which acts as a weighting or penalty function of the object pixels), resulting in successful pruning of minor branches and robust overall performance. Figure 3 shows examples of the vessel skeletonization algorithm.

The final step of our automation application is to calculate the Doppler angle for any arbitrary point, specified by the ultrasound system operator, within the vessel. For this purpose, we use the skeleton points around the specified vessel site of interest to fit a least squares line whose orientation is taken as the estimate of the vessel orientation (and therefore can be used to derive the Doppler angle) at this site. Figure 3 shows examples of Doppler angle estimation for arbitrary sites within different vessels.

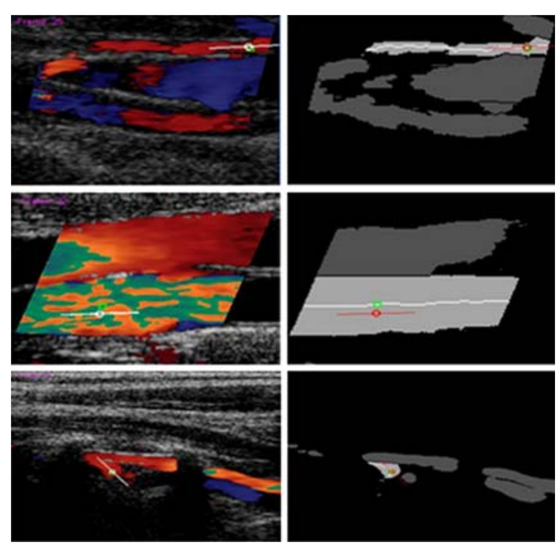

Fig 3. Vessel skeletonization and Doppler angle automation results. The *left column* shows the composite grayscale and color ultrasound image with the estimated angle as a *straight line*. The *right column* shows the segmented image with the skeleton of the vessel of interest as a *curve*, an arbitrary vessel site of interest as a *circle*, the nearest skeleton point to the vessel site as a *rectangle*, and the Doppler angle estimation as a *straight line*.

RESULTS

To verify the accuracy of the automatic Doppler angle estimation against the manual angle setting currently done by ultrasound system operators, we compared the automatically detected angles with the manual angles set by expert sonographers from Philips Ultrasound.

We conducted two types of experiments to quantify the accuracy of the angle estimation. In the first type of experiments, we developed a Matlab®-based off-line application to present color Doppler images to the sonographers and provide them with a simple user-interface tool to draw the Doppler angle line, manually aligning it with the vessel orientation according to their perception. Then, we calculated the angle automatically using our technique for the same vessel site and compare the two estimates. Figure 4 shows a screenshot of the off-line evaluation display; the dashed yellow line is manually drawn by the sonographer to represent the vessel's orientation at this site. The solid white line is the angle provided by the automation technique.

In the second type of experiments, the participating sonographers used a real-time prototype of the Doppler angle automation technique implemented on the Philips ultrasound system iU22 to scan volunteered human models in a real clinical setup. The sonographers conducted normal vascular exams by interrogating different vessels. First, they used the user-interface knob provided by the system to align the Doppler angle cursor with the vessel orientation. Then, they triggered the automation at the same vessel site to compare the results.

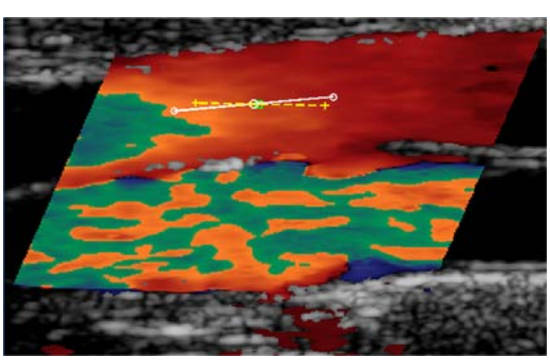

Fig 4. **Matlab®-based off-line evaluation of the automatic Doppler angle accuracy. The *dashed line* represents the manual specification of the Doppler angle. The *solid line* represents the automatic specification of the Doppler angle.**

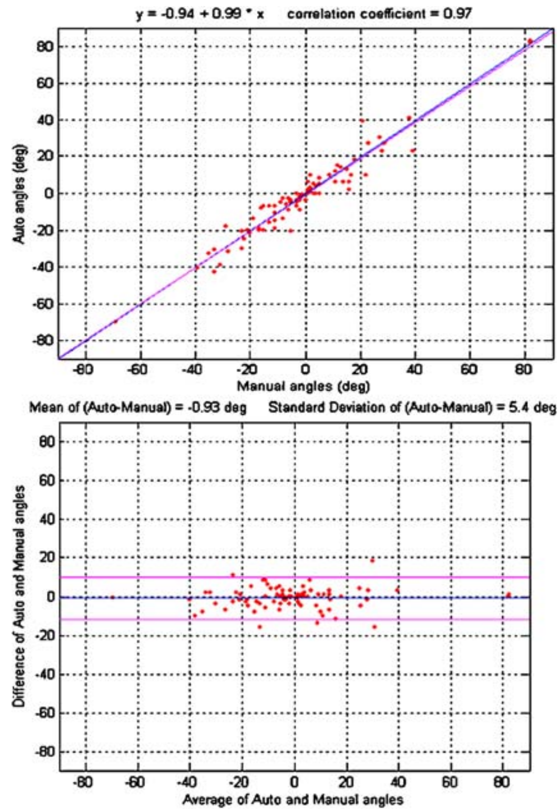

Fig 5. **Experimental results of the Matlab-based off-line evaluation. *Top* *x–y* scatter plot of automatic vs. manual angles, plus identity line, and least squares line. *Bottom* Bland–Altman plot of the difference of the auto and manual angles vs. the average of the auto and manual angles, plus mean of differences, and mean ± two standard deviations of differences.**

We collected data from both experiment types conducted by two different sonographers and analyzed the results as shown in Figures 5 and 6. Two display formats are presented: the *x–y* scatter plot, which is useful for appreciating the degree of correlation between the two types of measurements, and the Bland–Altman plot (difference of individual measurements vs. average of individual measurements), which is best for quantifying numerical agreement between two different methods[25]. For both types of experiments, the automatic and manual angle results are very similar, exhibiting high correlation coefficients (0.97 for the off-line experiment and 0.99 for the real-time prototype experiment), low means of individual differences (−0.93 degrees for the simulation experiment and 1.66 degrees for the real-time prototype experiment), plus reasonable standard deviations of the

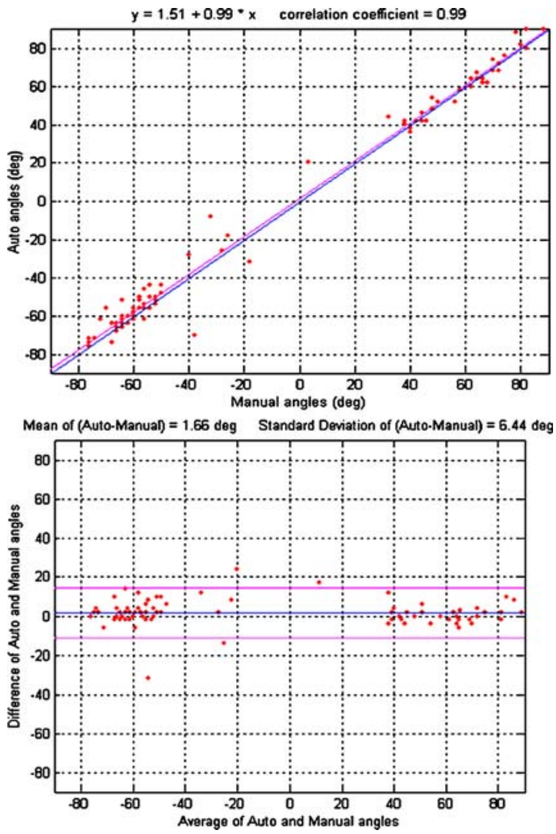

Fig 6. Experimental results of the real-time Doppler angle automation prototype. *Top x–y* scatter plot of automatic vs. manual angles, plus identity line, and least squares line. *Bottom* Bland–Altman plot of the difference of the auto and manual angles vs. the average of the auto and manual angles, plus mean of differences, and mean ± two standard deviations of differences.

individual differences (5.4 degrees for the simulation experiment and 6.44 for the real-time prototype experiment).

DISCUSSION

In this work, we have developed a computer vision approach to automate the Doppler angle estimation and replace the repetitive and sometimes inconsistent manual Doppler angle adjustments. The chosen angle greatly affects the accuracy of blood flow velocity measurements, which in turn can have a major effect on the diagnosis and grading of arterial stenosis.

The segmentation technique based on shape decomposition that was outlined above gave promising results with normal healthy vessels. However, unsatisfactory performance may be obtained in the context of challenging cases such as vessel over-segmentation due to poor blood filling, noisy color images with flash artifacts, and complex vessel geometry due to pathology. One enhancement could be to apply multiscale curvature-based shape evolution techniques to simplify the representation of the noisy vessel contours in ultrasound images[26,27]. Another enhancement could be combining gray-scale and color Doppler information to guide the segmentation. Through studying many ultrasound vessel image sequences, we observed that none of the two image types is ideal to represent the real vessel anatomy alone due to the inherent artifacts in each image type. We believe that through combining the useful information from both image types, we should be able to develop robust vessel segmentation techniques for ultrasound imaging.

The use of temporal information to guide the segmentation is also a promising direction since ultrasound imaging is a real-time modality. By using the correlation between image sequences, we should be able to track the vessel in every frame. We have already used the temporal information by averaging a number of frames to obtain a pulsatility-independent representation of blood flow within the vessel of interest. However, more advanced techniques can be developed to deal with motion artifacts (due to patient breathing or transducer movement) through the introduction of motion estimation techniques. Also, scene change detection techniques can be applied to filter outlier frames (due to movement) before averaging.

The angle estimation technique using the least squares line fitting of the skeleton points gave reasonable results in most cases. However, the estimated angle is not accurate if the vessel is severely curved or tortuous since the line fit may be considering points that do not contribute to the vessel orientation at this specific site. One enhancement could be to adaptively select a variable number of skeleton points to contribute to the line fittings based on minimizing the fitting error. Another enhancement could be to fit a higher-order polynomial or a spline to the adaptively selected skeleton points. Finally, an alternative approach would be to approximate the Doppler angle as the Discrete Tangent Orientation of the skeleton curve. There are many techniques in the literature to estimate the discrete tangent orientation[28].

It is worth mentioning that the effect of the difference between the manual and automatic angles on the velocity estimation (which is the diagnostic criterion used to grade vessel stenosis) will be further minimized by the fact that the velocity relies on the cosine of the angle not the angle itself.

There are two main approaches for manual Doppler angle correction: parallel to the vessel wall versus parallel to the blood flow[13]. A survey of ICAVL-accredited laboratories was undertaken and reported by Madrazo[29], indicating that approximately 72% of accredited laboratories were utilizing the diagnostic criteria published by the Strandness et al.[1] from the University of Washington. The guidelines established by the University of Washington for those specific criteria include using a consistent angle of 60° and with the Doppler adjusted parallel to the vessel wall. Our approach only uses color Doppler data to automate the angle correction, which seems similar to the second approach (parallel to the blood flow). For the automation purpose, the use of color Doppler data should offer increased robustness than relying on just the B-mode images since B-mode images typically exhibit fuzzy and incomplete vessel walls. If there is good color Doppler filling within the vessel, then the representative averaged frame, which is calculated from a number of color Doppler frames, will match the vessel wall to a great extent.

One potential limitation of the approach described here is with severe stenotic cases where the streamline of the blood flow may depart from the vessel center line for a short distance after a sharp bend or narrowing plaque. In these cases, the detected angle may not match the streamline since the algorithm relies on the properties of the detected color flow Doppler boundaries, not the profile of velocity within the color flow channel. The exact relevance of this potential limitation can only be established by carefully planned in vitro experiments plus large-scale clinical trials. One important mitigation method for such extreme cases is the user ability to override the automated angle if it seemed off. In addition, more sophisticated angle estimation techniques can be developed for such extreme cases.

One important aspect of such automation features, which is as important as the algorithm performance and robustness, is the user interface. How to integrate the automation features with the current clinical workflow is a critical question. We have considered two main aspects related to the user interface; the first is the trigger mechanism of the angle automation, and the second is the interaction with the manual angle adjustment. We have investigated two trigger mechanisms; the first mechanism allows the user to manually trigger the angle automation when needed. The second mechanism implements an automated triggering of the angle adjustment based on the Doppler gate movement over the underlying vessel. The manual-trigger mechanism gives the user full control over when and where to activate the angle adjustment, while the automatic-trigger mechanism saved the user multiple button clicks for the different angle adjustments during the Doppler exam. Sonographers involved in the clinical experiments found the two triggering mechanisms useful in different situations and preferred to have the ability to setup the machine to choose one of the two triggering mechanisms. The interaction with the existing Doppler angle adjustment control was a critical user-interface decision as well. The involved sonographers preferred to maintain the ability to override the automated angle using the manual control in case of inaccurate or wrong results. Further experiments with broader number of users are necessary for concluding the final user interface of this automation feature.

CONCLUSIONS

In this paper, we have outlined a computer vision approach for automatically estimating the Doppler angle, which is a critical component of the accurate quantification of blood flow by Doppler ultrasound. The developed automation technique involves multiple processing steps aiming to successfully deal with imaging artifacts and other performance issues, plus patient- and/or pathology-driven data complexity.

The initial experience obtained from an off-line automation prototype was very encouraging and justified the development of a real-time prototype, which was fully integrated within a Philips Ultrasound iU22 scanner. We have performed a preliminary quantitative evaluation of the accuracy of the automated Doppler angle estimates vs. manual measurements performed by expert observers and have demonstrated good agreement for off-line, as well as real-time, experiments, However, we have also observed that performance of the automation technique may be degraded when applied to data of poor image quality or very complex nature.

The next step for our automation technique is to deploy the real-time prototype in a real clinical environment and assess its effect on the vascular exam workflow and diagnostic quality. Evaluating the automation technique in controlled clinical trials should prove its real clinical benefits and merits and also clearly identify algorithmic aspects that need to be improved.

Overall, the experience accumulated so far strongly suggests that the Doppler angle automation technique presented here can prove very helpful in streamlining the ultrasound Doppler vascular exam and achieving more accurate and standardized outcomes. If pursued thoroughly, such automation approaches have the potential—apart from the obvious ergonomic and time-related benefits—to also enhance the quality of the patient diagnosis and treatment decisions.

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