CSE 531: Complexity theory | Winter 2007
Problem Set #2 | Instructor: Venkatesan Guruswami
Due in class on Monday, **February 5, 2007**.

Instructions: Same as for Problem set 1.

Each problem is worth 10 points unless noted otherwise.

1. Define the language

$$MAX2SAT = \{\langle \phi, k\rangle \mid \phi \text{ is a 2CNF Boolean formula and there exists an assignment that satisfies at least } k \text{ clauses of } \phi\} .$$

   Prove that MAX2SAT is NP-complete.

2. We know that the language

$$2SAT = \{\langle \phi \rangle \mid \phi \text{ is a satisfiable 2CNF formula}\}$$

   is in P. You are now required to show that 2SAT is NL-complete. (Do not forget to show that 2SAT $\in$ NL.)

3. We showed in class that $\mathrm{ALL_{NFA}}$ is in PSPACE. In this problem, you are required to prove that $\mathrm{ALL_{NFA}}$ is in fact PSPACE-complete.

   (Hint: Come up with a regular expression (in polynomial time) that generates all strings that are not rejecting computations histories of a polynomial space TM M on input w. The proof of Theorem 9.15 in Sipser's book should help you figure out how to do this.)

4. An *unrestricted grammar* (or a *rewriting system*) is a 4-tuple $G = (V, \Sigma, R, S)$ where

   - V is an alphabet;
   - $\Sigma \subset V$ is the set of *terminal* symbols, and $V - \Sigma$ is called the set of *nonterminal* symbols;
   - $S \in V - \Sigma$ is the *start* symbol; and
   - R, the set of *rules*, is a finite subset of $(V^*(V - \Sigma)V^*) \times V^*$.

   (Thus the "only" difference from context-free grammars is that the left-hand sides of rules need not consist of single nonterminals.) Let us write $\alpha \to \beta$ if $(\alpha, \beta) \in R$; and let's define $u \Rightarrow_G v$ iff, for some $w_1, w_2 \in V^*$ and some rule $\alpha \to \beta \in R$, $u = w_1\alpha w_2$ and $v = w_1\beta w_2$. Let $\overset{*}{\Rightarrow}_G$ denote the reflexive, transitive closure of $\Rightarrow_G$. We say that a string $w \in \Sigma^*$ is generated by G if and only if $S \overset{*}{\Rightarrow}_G w$. Finally, $L(G) \subseteq \Sigma^*$, the *language generated by* G, is defined to be the set of all strings in Σ^* generated by G.

   Now, define a context-sensitive grammar to be one for which whenever $(x, y) \in R$ we have $|x| \le |y|$ (i.e., the right hand side of rules are at least as long as the left hand size). Now to your exercises.

   Prove that the class of languages generated by context-sensitive grammars is precisely NSPACE(n).

   (Hint: The harder direction is to prove that languages in NSPACE(n) are context-sensitive. For this, it might help to construct a grammar whose rules simulate backward moves of

M (for an arbitrary TM M), and whose derivations will consequently simulate backward computations of M. This will show that unrestricted grammars can generate any Turing recognizable language. Now see how the space restriction can be used to argue that the grammar may be assumed to be context-sensitive.)

5. Define a language to be k-shallow if it is accepted by a family of depth two circuits that consist of an AND of OR's where the fan-in of each OR gates is bounded by a constant k, independent of the length n of the input. Prove that if both A and the its complement are k-shallow, then membership in A can be tested by examining a constant number (independent of n) of input positions.

   (Hint: One approach is to use induction on k.)