CSE 531: Computability and Complexity
Assignment #6
November 15, 2001
due: Tuesday, November 27

In all problems, you should assume that integers are given in binary, vertices in a graph $G = (V, E)$ are listed in the input as the binary numbers from 1 to $|V|$, and E is given as a list of pairs of such binary numbers.

1. Problem 7.21.

2. Problem 7.22. (Hint: somewhere in part (b) it will be helpful to use the result of part (a).)

3. Problem 7.23. Warning: the k in the definition of the language $MAX-CUT$ is not the same k used later in the problem, the number of clauses. To avoid confusion, change k to k' in the definition of the language.

   Continuation of the book's hint: Suppose that you have a cut in which the $3k$ nodes labeled x are split between S and T, say i of them in S and $3k-i$ of them in T, where $1 \leq i \leq 3k/2$. Then I claim that at least $3k$ of the edges connecting nodes labeled x and nodes labeled $\overline{x}$ are not in the cut. Here is the reason: suppose j nodes labeled $\overline{x}$ are in T and the remaining $3k - j$ are in S. Then the number of $x - \overline{x}$ edges not in the cut is

$$\begin{aligned} i(3k-j) + j(3k-i) &= 3ki + 3kj - 2ij \\ &\geq 3k \cdot 1 + 3kj - 2j(3k/2) \\ &= 3k. \end{aligned}$$