CSE 531: Computability and Complexity
Assignment #5
November 8, 2001
due: Thursday, November 15

In all problems, you should assume that integers are given in binary, vertices in a graph $G = (V, E)$ are listed in the input as the binary numbers from 1 to $|V|$, and E is given as a list of pairs of such binary numbers.

1. (a) Prove that $\leq_L$ is transitive. Be sure that your Turing machine only uses $O(\log n)$ worktape space for all intermediate results.

   (b) Suppose that A is NL-complete, $B \in NL$, and $A \leq_L B$. Prove that B is NL-complete.

2. Let $\mathit{NFA}\varepsilon = \{< M >|\ M \text{ is an NFA and } \varepsilon \in L(M)\}$. Prove that $\mathit{NFA}\varepsilon$ is NL-complete. (This strengthens Exercise 8.8. Hint: given Problem 1 above, it suffices to prove that $\mathit{NFA}\varepsilon \in NL$ and $PATH \leq_L \mathit{NFA}\varepsilon$.)

3. Let $SPATH$ be the language given in problem 7.16. Prove that $SPATH$ is NL-complete. (Hint: given Problem 1a from Assignment 4 and Problem 1 above, it suffices to prove that $PATH \leq_L SPATH$. If $G = (V, E)$ is the graph input to $PATH$, consider a graph with $|V|$ copies of the vertex set V. You'll have to figure out what undirected edges to put in this graph, and what values of a, b, and k to use. Be sure to include a proof of correctness of your mapping reduction, and a proof that the reduction only uses $O(\log n)$ space.)

4. Let
$$CYCLE = \{< G >|\ G \text{ is a directed graph with a directed cycle}\}.$$
   Prove that $CYCLE$ is NL-complete. (Hint: see the hint for Problem 3.)