CSE 531: Computability and Complexity
Assignment #4
October 30, 2001
due: Tuesday, November 6

In all problems, you should assume that integers are given in binary, vertices in a graph $G = (V, E)$ are listed in the input as the binary numbers from 1 to $|V|$, and E is given as a list of pairs of such binary numbers.

1. Let *SPATH* be the language given in problem 7.16.
   (a) Show that $SPATH \in NL$.
   (b) Now do problem 7.16(a). Hint: This part is going to be worth far fewer points than part (a) above.
2. Show that the language MAX-CUT of Problem 7.23 is in NP. Determine and explain a precise upper bound on the running time of your NTM.
3. Let M be a Turing machine, either deterministic or nondeterministic. Suppose that M has space complexity $s(n) \geq \log_2 n$. What is the maximum running time of M as a function of $s(n)$? Justify your answer. (Hint: Recall that M must halt on all computation branches and all inputs.)
4. Let

   $$UPATH = \{< G, s, t >|\ G \text{ is an undirected graph with a path between vertices } s \text{ and } t\}.$$

   There is lots of evidence to suggest that $UPATH \in L$, and if I had to bet I would bet it is so. Make as much progress as you can in resolving this question. One possibility would be to show that it is true for various special types of graphs. Another possibility would be to show that some related undirected graph traversal problems are in L.

   I would be happy to hear any interesting ideas you come up with, even after the due date of this assignment.