CSE 521: The Design and Analysis of Algorithms
Assignment #3
April 28, 1997
due: Wednesday, May 7

1. Exercise 16.3-6 on page 319. Support your algorithm with a sketch of correctness and time analysis. (If you cannot solve this exercise, substitute 16.3-5.)

2. In this problem you will devise an algorithm for a variant of alignment called "local alignment" that often concerns biologists. Given two strings S and T, the local alignment problem is to find (contiguous) substrings A of S and B of T such that the optimal (ordinary) alignment of A and B has the maximum value, over all such substrings A and B. For instance, if a match scores $+2$ and a mismatch, insertion, or deletion scores -1, then an optimal local alignment of abcxdex and xxxcde is $\begin{matrix} c & x & d & e \\ c & - & d & e \end{matrix}$.

   Devise an $O(nm)$ time algorithm, where $n = |S|$ and $m = |T|$. You may assume that $\sigma(x, -) \leq 0$ and $\sigma(-, x) \leq 0$. (Hint: let $v(i, j)$ be the maximum value of an optimal (ordinary) alignment of α and β, over all suffixes α of $S[1..i]$ and all suffixes β of $T[1..j]$.)

3. Prove that the greedy algorithm for the fractional knapsack problem outputs the optimal solution. Do this as follows: by induction on the number of items considered, show that the partial solution constructed can be embedded in an optimal solution.

4. Exercise 17.2-6 on page 337. For the knapsack problem, you should assume that $W > 0$ and each $w_i > 0$. (Hints: First show how you can add one item to the knapsack problem so that $\sum_i w_i = 2W$, yet you can recover the optimal solution to the original instance from an optimal solution to this modified instance. The "black box" you will need is Problem 10-2(c). In that problem, change the names of the weights from w_i to ϵ_i to avoid confusion with the weights of the knapsack problem. Also, you may assume that the black box algorithm works as stated even when $x_1, x_2, \ldots, x_n$ are not necessarily all distinct.)

5. Exercise 17.4-4 on page 350.