CSE 521: Design and Analysis of Algorithms Assignment #6

due: Tuesday, Nov 22. 10:30AM

Each problem is worth 10 points. “Give an algorithm” means pseudo-code, a high-level explanation and a proof of correctness. See the website for more grading guidelines.

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1. Suppose you are given a set P of m points in $\mathbb{R}^n$, each labeled either black or white. A linear classifier is a pair $c \in \mathbb{R}^n, \theta \in \mathbb{R}$ with the following properties:

   - If p is a black point, then $c^T p > \theta$.
   - If p is a white point, then $c^T p < \theta$.

   Describe an efficient algorithm to find a linear classier for the given data points, or correctly report that none exists.

2. Suppose you have a subroutine that can solve linear programs in polynomial time, but only if they are both feasible and bounded. Describe an algorithm that solves arbitrary linear programs in polynomial time, using this subroutine as a black box. Your algorithm should return an optimal solution if one exists; if no optimum exists, your algorithm should report that the input instance is UNBOUNDED or INFEASIBLE, whichever is appropriate.

3. Given points $((x_1, y_1), \ldots, (x_n, y_n))$ in the plane, the linear regression problem asks for real numbers a and b such that the line $y = ax + b$ fits the points as closely as possible, according to some criterion. The most common t criterion is minimizing the L_2 error, defined as follows:

$$\epsilon_2(a,b) := \sqrt{\sum_{i=1}^{n} (y_i - ax_i - b)^2}.$$

   Such a fit can be found by solving a linear system of equations. But other fit criteria give rise to optimization problems that can be solved by linear programming.

   (a) The L_1 error (or total absolute deviation) of the line $y = ax + b$ is dened as follows:

$$\epsilon_1(a,b) := \sum_{i=1}^{n} |y_i - ax_i - b|.$$

   Describe a linear program whose solution (a, b) describes the line with minimum L_1 error.

   (b) The L_∞ error (or maximum absolute deviation) of the line $y = ax + b$ is dened as follows:

$$\epsilon_\infty(a,b) := \max_{i \in [n]} |y_i - ax_i - b|.$$

   Describe a linear program whose solution (a, b) describes the line with minimum L_∞ error.

4. Exercise 7.28 from DPV (Dasgupta, Papadimitriou and Vazirani), available online at `http://bit.ly/wbWER`