

due: Tuesday, Oct 18. 10:30AM

Each problem is worth 10 points. KT refers to *Algorithm Design*, First Edition, by Kleinberg and Tardos. “Give an algorithm” means pseudo-code, a high-level explanation and a proof of correctness. See the website for more grading guidelines.

1. KT, Chapter 4, Problem 14.
2. KT, Chapter 4, Problem 19. *Hint: It may help to first solve the problem in the case when all bandwidths are different.*
3. Given an undirected graph (V, E) , the cographic matroid (S, \mathcal{I}) is defined with $S = E$ and $X \subset S$ independent if $E \setminus X$ is connected. Prove that this is indeed a matroid.
4. Given an undirected graph $G = (V, E)$, the matching matroid (S, \mathcal{I}) is defined with $S = V$ and $X \subseteq S$ independent if there is a matching in G that covers X . (A matching is a subset of edges that is incident upon each vertex at most once.)
 - (a) Prove that this is a matroid. *Hint: The symmetric difference of sets A, B is $A \Delta B := (A \cap \bar{B}) \cup (\bar{A} \cap B)$. First determine what the symmetric difference of two matchings looks like.*
 - (b) Extra credit: Give an efficient algorithm to test whether a set is independent in the matching matroid. Using this, describe the performance of the resulting greedy algorithm.