CSE 521
Assignment 9
Due Tuesday, June 3, 2003

1. In this problem we will examine several on-line algorithms for list access: MF (move-to-front), T (transpose), and FC (frequency count) on a specific request sequence. Consider a list $x_1, x_2, \ldots x_k$ with the following sequence k accesses to x_k, $k-1$ accesses to x_{k-1}, all the way to 1 access to x_1. Altogether there are $n = k(k+1)/2$ accesses with no insertions or deletions.

   (a) Calculate (as a function of k) the cost of MF, T, and FC for this request sequence.

   (b) Use your result to prove that T and FC are not constant competitive. Use the fact that MF is 2-competitive to achieve your result.

2. A generalization of the paging problem is called the k-server problem where we have a metric space (M, d) of points and k servers which lie on k points. Recall a metric space (M, d) has the property that d is a mapping from $M \times M$ into the real numbers such that $d(x, y) \geq 0$, $d(x, y) = 0$ implies $x = y$, $d(x, y) = d(y, x)$, and $d(x, z) \leq d(x, y) + d(y, z)$. A request is simply a member of M. A request is said to be served if one of the servers is on the requested point. The cost of serving a request r is $d(r, x)$ where the server on point x is moved to r to serve the request.

   (a) Define a metric space that makes the paging problem with cache size k into a k-server problem.

   (b) Consider the following metric space with exactly three points a, b and c on a line. The points a and c are 1 unit apart and b is between a and c exactly with distance 1/3 from a and 2/3 from c. There are 2 servers. The *greedy on-line algorithm* always serves a request by moving the nearest server to it. Show that the greedy on-line algorithm is not constant competitive.