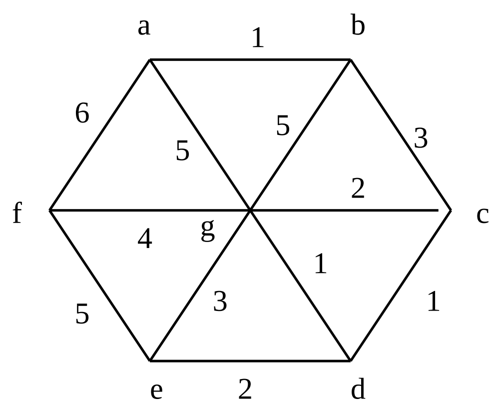
CSE 589
Final Exam
December 19, 2001

1. (10%) Use the LZW decoding algorithm to encode the sequence ababcabd where the initial dictionary is

0	a
1	b
2	c
3	d

   Show the resulting dictionary that the encoder generates.

2. (10%) Use Kruskal's algorithm to find the minimum spanning tree for the following graph. List the edges in the order they are considered for inclusion into the minimum spanning tree.



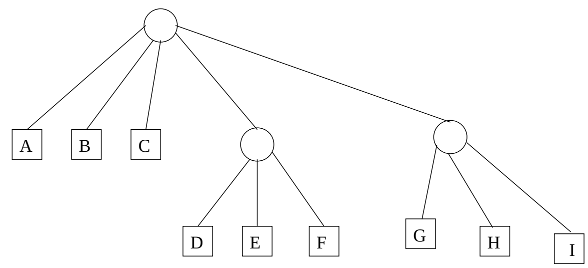
3. (15%) Suppose the symbols $\{a, b, c\}$ have the following probabilities,

   $a : 1/3$, $b : 1/6$, $c : 1/2$.

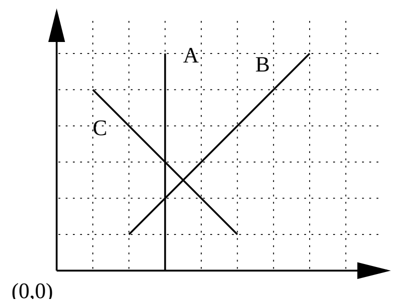
   (a) Design the optimal Huffman code for these symbols taken **two** at a time. That is, the symbols are grouped into $aa, ab, ac, ba, bb, bc, ca, cb, cc$. Show the resulting Huffman tree.

   (b) What is its bit rate?

4. (15%) Use the PQ tree algorithm to find the restricted PQ tree that has the set $\{B, C, E, F, G\}$ contiguous. Show the steps in the algorithm pictorially.



5. (15%) Consider the following lines $A = [(3,6),(3,0)], B = [(7,6),(2,1)], C = [(1,5),(5,1)]$ as shown below.



   (a) Give the initial event queue for the plane sweep intersection finding algorithm.

   (b) Run the plane sweep intersection finding algorithm showing the event queue and segment list at each step.

6. (10%)

   (a) Explain briefly how bit plane encoding works in modern wavelet based image compression algorithms like SPIHT, GTW, and JPEG 2000.

   (b) Briefly explain why the SPIHT image compression algorithm is a kind of group testing algorithm.

7. (15%) Consider the problem of finding a *longest increasing subsequence* of a sequence $x_1, x_2, \ldots, x_n$ of positive numbers. We define $s_1 < s_2 < \cdots < s_k$ to be an increasing subsequence of $x_1, x_2, \ldots, x_n$ of length k if there is a sequence $i_1 < i_2 < \cdots < i_k$ such that $s_j = x_{i_j}$ for $1 \leq j \leq k$. For example 2,4,5 is an increasing subsequence of 4,2,4,2,5,2 by taking the 2nd, 3th, and 5th numbers.

   Given $x_1, x_2, \ldots, x_n$ and $i \geq j$, define $M[i, j]$ to be the length of a longest increasing subsequence of $x_1, x_2, \ldots, x_i$ that ends in x_j.

   (a) What is the value of $M[i, 1]$ for $1 \leq i \leq n$?

   (b) For $j \leq i$ give a recursive definition of $M[i, j]$ in terms of $M[i-1, 1]$, $M[i-1, 2], \ldots, M[i-1, i-1]$.

   (c) Briefly describe an $O(n^2)$ dynamic program to find the length of the longest increasing subsequence using (a) and (b).

   (d) Continue your description to explain how to find the actual longest increasing subsequence.

8. (10%) Please answer true or false.

   (a) The asymptotic analysis of an algorithm does not depend on the data structure used in the algorithm.

   (b) An amortized complexity bound gives a bound on the average time per operation over a worst case sequence of operations.

   (c) Branch-and-bound algorithms achieve a global optimum.

   (d) Problems that on the surface seem to require an exponential search are NP-hard.

   (e) Finding a minimum spanning tree in an undirected labeled graph is NP-hard.

   (f) First order entropy is a lower bound on the number of bits to compress any string by an algorithm that only uses symbol frequencies and no other information.

   (g) Image compression based on the wavelet transform suffers from blocking artifacts in compressed images.

   (h) The SPIHT image compression algorithm does not need arithmetic coding to enhance its performance.

   (i) The convex hull of n points in two dimensions can be found in linear time.

   (j) Exclusion search is always faster, in order of magnitude, than dynamic programing for approximate string searching.