

Section 2 – Link Layer

CSE 461 – Autumn 2015

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Byte Count

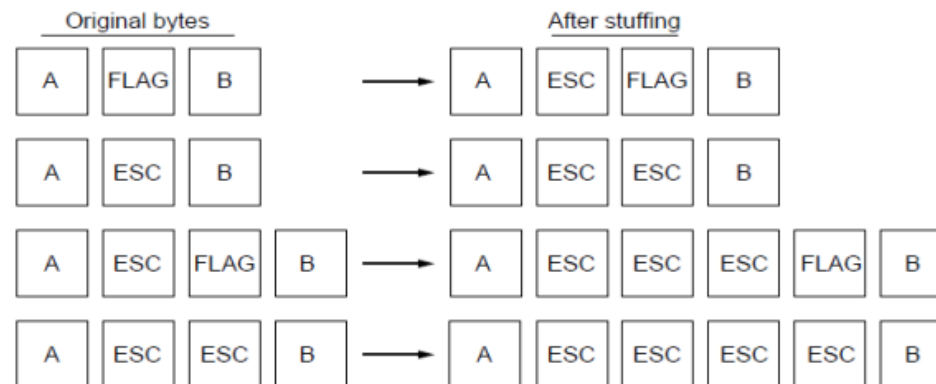
- Add a length to the start of the frame
- No protection against any errors

Byte Stuffing

- Have a special flag byte value that means start/end of frame

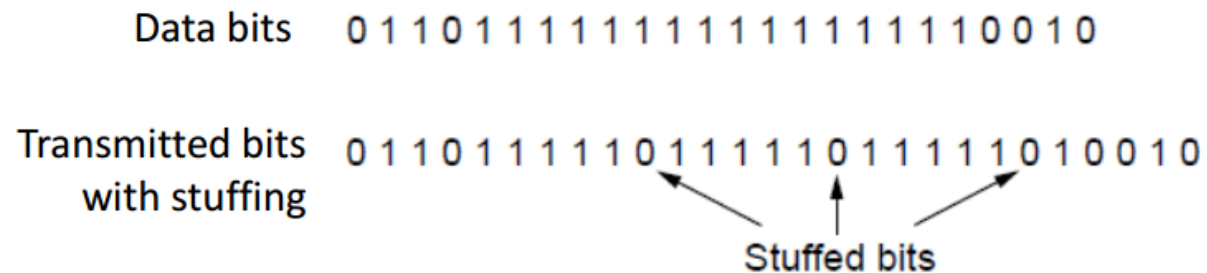


- Replace the flag inside the frame with an escape code



Bit Stuffing

- Like byte stuffing but in the bit level
- Use six consecutive 1s as the flag
 - On transmit, after five 1s in the data, insert a 0
 - On receive, a 0 after five 1s is deleted



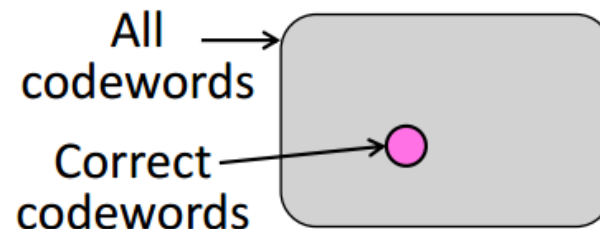
Error Detection and Correction

- Done with check bits, calculated from the data to be transmitted
- More check bits usually means more errors can be detected and calculated
- However, it's a balance between the overhead of check bits and the reliability from those check bits



Why Check Bits Work

- The combination of the data and check bits can be called a codeword
- The check bit works because there's a lot more codewords than valid ones (the check bits matches the check bits calculated from the data)
- So it's very unlikely that errors can transform a valid codeword into a different valid codeword



Hamming Distance

- Distance is the number of bit flips needed to change $D1$ to $D2$
- Hamming distance of a code is the minimum distance between any pair of valid codewords
- For a code of distance $d+1$, up to d errors will always be detected
- For a code of distance $2d+1$, up to d errors can always be corrected by mapping to the closest codeword

Error Detection

- Standard functions to create the check bits:
 - Parity bit, 1 check bit from the sum of all data bits, Hamming distance of 2
 - Checksum, 16 check bits from 16-bit ones complement arithmetic, Hamming distance of 2, good for Burst Errors
 - CRC (Cyclic Redundancy Check), k check bits from n data bits such that n+k bits are evenly divisible by a generator C, Hamming distance of 4, good for Burst Errors up to k bits

Checksum

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
  0001
  f203
  f4f5
  f6f7
+ (0000)
-----
 2ddf0
  ↓
  ddf0
+    2
-----
  ddf2
  ↓
 220d
```

Receiving:

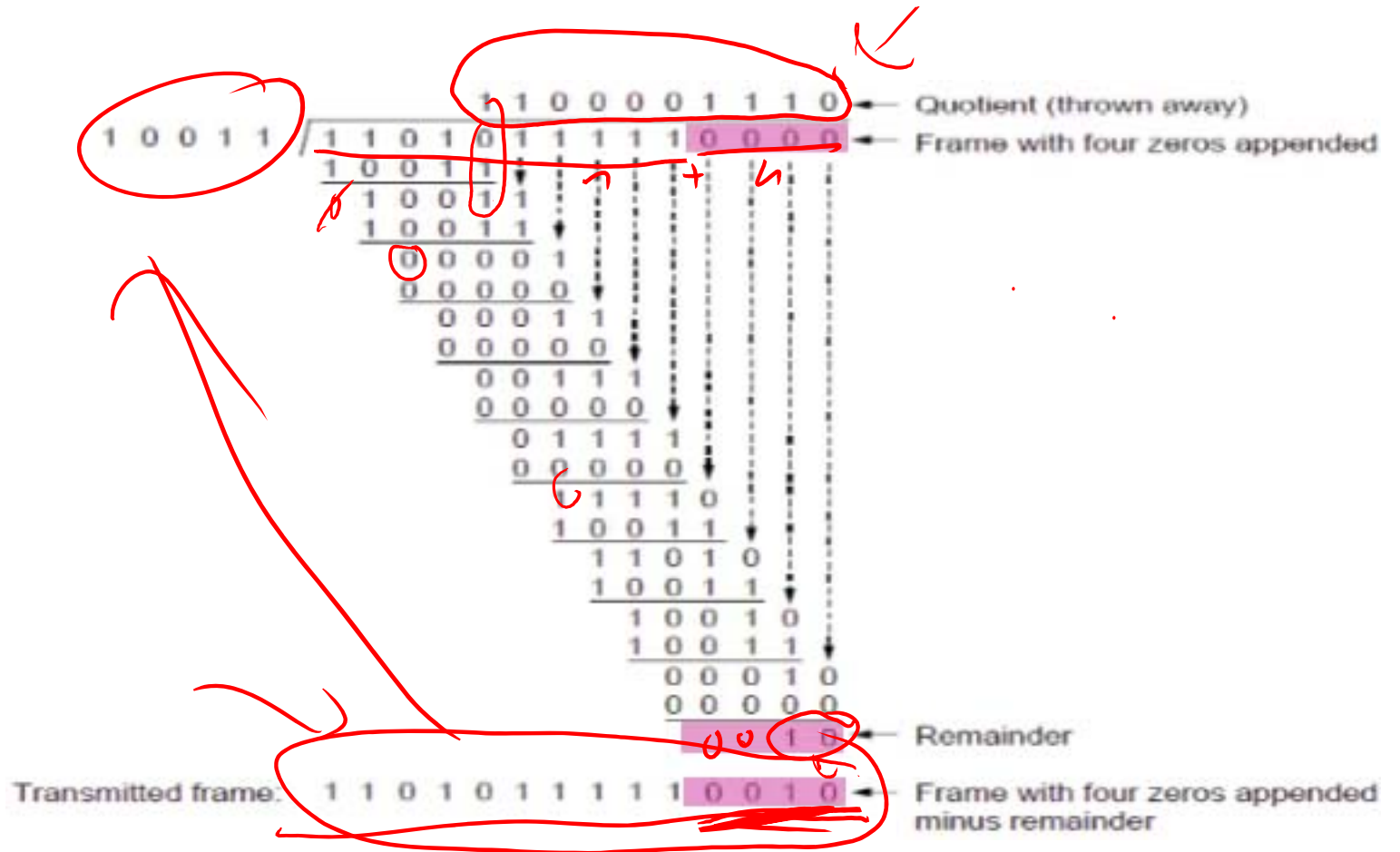
1. Arrange data in 16-bit words
2. Checksum will be non-zero, add
3. Add any carryover back to get 16 bits
4. Negate the result and check it is 0

```
  0001
  f203
  f4f5
  f6f7
+ 220d
-----
 2fffd
  ↓
  fffd
+    2
-----
  ffff
  ↓
 0000
```

CRC

Data bits:
110101111

Check bits:
 $C(x) = x^4 + x^1 + 1$
 $C = 10011$
 $k = 4$



Error Correction

- Harder than detection, can correct only d errors in codewords with Hamming distance $\geq 2d + 1$
- In this class we will mostly talk about Hamming Code for error correction

Hamming Code

- Allows the creation of a codeword with a Hamming distance of 3, for every n data bits there must be k check bits where $(n = 2^k - k - 1)$
- The check bits are located in positions that are powers of 2, so $1 = 2^0$, $2 = 2^1$, $4 = 2^2$, etc.
- Check bits in position p is parity for positions with a p term in their values

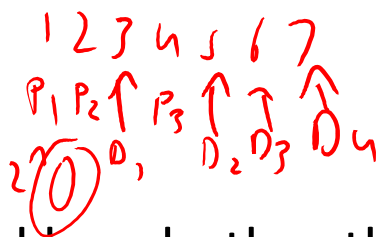
Data 1010

Hamming Code Check Bits Coverage

Data = 4 bits, Check bits = 3 bits, Codeword = 7 bits

Check bits are located at:

- $1 = 2^0$, which means they cover 3, 5, & 7
- $2 = 2^1$, which means they cover 3, 6, & 7
- $4 = 2^2$, which means they cover 5, 6, & 7



Position

| Decimal | Binary |
|---------|--------|
| 1 | 001 |
| 2 | 010 |
| 3 | 011 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |

What the check bits cover are determined by whether the location contains them in their term or in other words, the location in binary has a 1 at the check bit's power to 2.

The value of the check bits themselves are the summation of the bits at those positions.

$P_1 = 0_3 \wedge 0_5 \wedge 0_7$

Hamming Code Example

$$p_1 \oplus D_3 \oplus D_5 \oplus D_7$$

To decode:

- Recompute check bits (with parity sum including the check bit)
- Arrange as a binary number
- Value (syndrome) tells error position
- Value of zero means no error
- Otherwise, flip bit to correct

$$p_1 \rightarrow \begin{array}{ccccccc} \underline{0} & \underline{1} & 0 & \underline{0} & 1 & \mathbf{1} & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$

$$p_1 = 0 + 0 + 1 + 1 = 0, \quad p_2 = 1 + 0 + \mathbf{1} + 1 = \mathbf{1},$$
$$p_4 = 0 + 1 + \mathbf{1} + 1 = \mathbf{1}$$

Syndrome = $\mathbf{110}$, flip position 6

Data = 0 1 0 1 (correct after flip!)

Error Detection vs. Correction

- Usually error correction is used when errors are expected and there's no time to retransmit
- While error detection is more efficient when errors are not expected or when the errors are really large so no hope of correction anyway
- But to choose one or the other still depends on the amount of data being sent and the rate of error