Error Detection

- Some bits may be received in error due to noise. How do we detect this?
  - Parity »
  - Checksums »
  - CRCs »

- Detection will let us fix the error, for example, by retransmission (later).

Simple Error Detection – Parity Bit

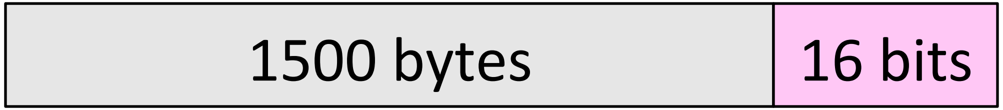
- Take D data bits, add 1 check bit that is the sum of the D bits
  - Sum is modulo 2 or XOR

Parity Bit (2)

- How well does parity work?
  - What is the distance of the code?
  - How many errors will it detect/ correct?
- What about larger errors?

Checksums

- Idea: sum up data in N-bit words
  - Widely used in, e.g., TCP/IP/UDP



- Stronger protection than parity

Internet Checksum

- Sum is defined in 1s complement arithmetic (must add back carries)
  - And it's the negative sum
- *"The checksum field is the 16 bit one's complement of the one's complement sum of all 16 bit words …"* – RFC 791

Internet Checksum (2)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
0001
f203
f4f5
f6f7
```

Internet Checksum (3)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
   0001
   f203
   f4f5
   f6f7
+(0000)
------
  2ddf0
    ↓
   ddf0
+     2
------
   ddf2
    ↓
   220d
```

Internet Checksum (4)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add

```
  0001
  f203
  f4f5
  f6f7
+ 220d
------
```

3. Add any carryover back to get 16 bits

4. Negate the result and check it is 0

Internet Checksum (5)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add
3. Add any carryover back to get 16 bits
4. Negate the result and check it is 0

```
   0001
   f203
   f4f5
   f6f7
+  220d
 ------
  2fffd
    ↓
   fffd
+     2
 ------
   ffff
    ↓
   0000
```

Internet Checksum (6)

- How well does the checksum work?
  - What is the distance of the code?
  - How many errors will it detect/ correct?
- What about larger errors?

Cyclic Redundancy Check (CRC)

- Even stronger protection
  - Given n data bits, generate k check bits such that the n+k bits are evenly divisible by a generator C
- Example with numbers:
  - n = 302, k = one digit, C = 3

CRCs (2)

- The catch:
  - It's based on mathematics of finite fields, in which "numbers" represent polynomials
  - e.g, 10011010 is $x^7 + x^4 + x^3 + x^1$
- What this means:
  - We work with binary values and operate using modulo 2 arithmetic

CRCs (3)

- Send Procedure:

1. Extend the n data bits with k zeros
2. Divide by the generator value C
3. Keep remainder, ignore quotient
4. Adjust k check bits by remainder

- Receive Procedure:

1. Divide and check for zero remainder

CRCs (4)

Data bits:

1101011111

Check bits:

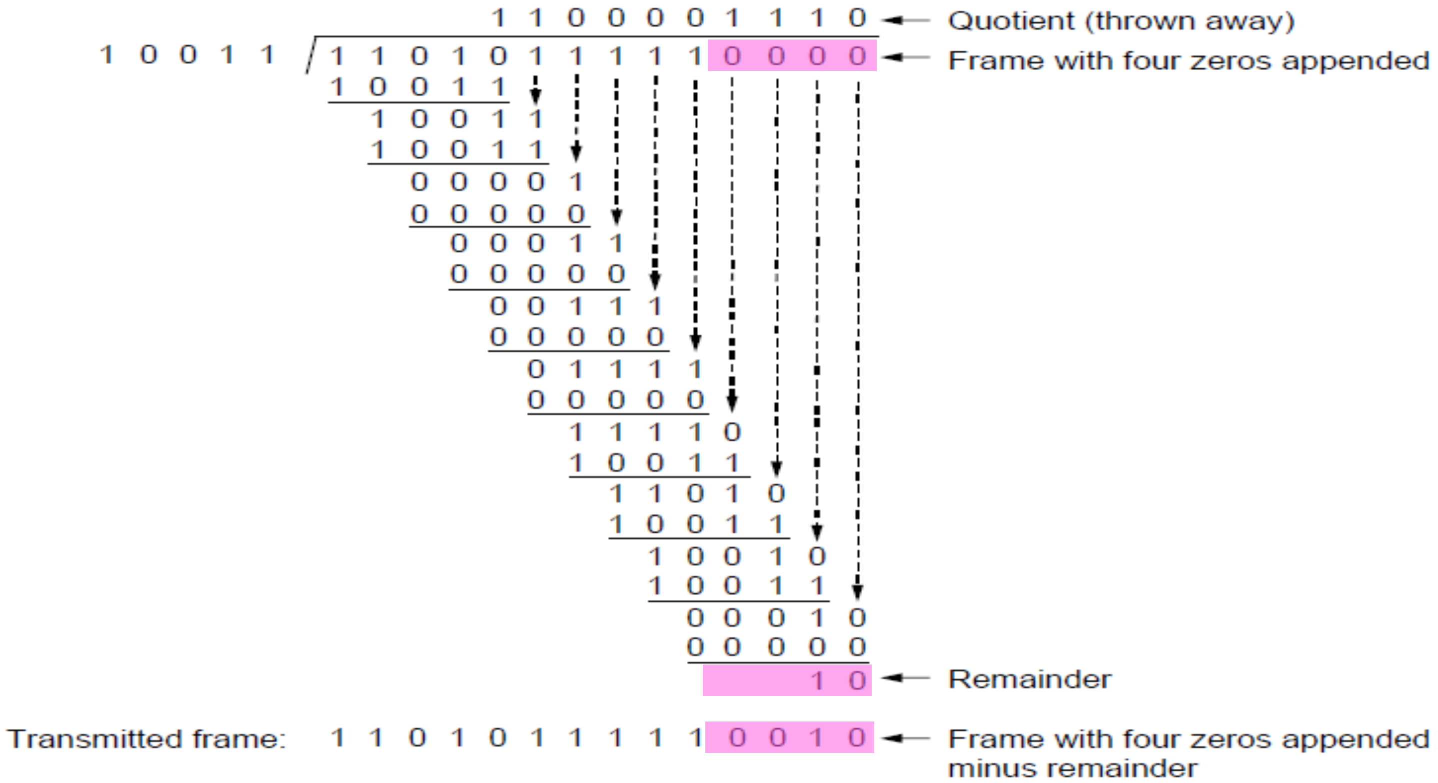
$C(x)=x^4+x^1+1$

C = 10011

k = 4

$$10011 \overline{)\,1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1\ 1}$$

CRCs (5)


1 1 0 0 0 0 1 1 1 0
Quotient (thrown away)
1 0 0 1 1
1 1 0 1 0 1 1 1 1 1 0 0 0 0
Frame with four zeros appended
1 0 0 1 1
1 0 0 1 1
1 0 0 1 1
0 0 0 0 1
0 0 0 0 0
0 0 0 1 1
0 0 0 0 0
0 0 1 1 1
0 0 0 0 0
0 1 1 1 1
0 0 0 0 0
1 1 1 1 0
1 0 0 1 1
1 1 0 1 0
1 0 0 1 1
1 0 0 1 0
1 0 0 1 1
0 0 0 1 0
0 0 0 0 0
1 0
Remainder
Transmitted frame: 1 1 0 1 0 1 1 1 1 1 0 0 1 0
Frame with four zeros appended minus remainder

CRCs (6)

- Protection depend on generator
  - Standard CRC-32 is 10000010 01100000 10001110 110110111
- Properties:
  - HD=4, detects up to triple bit errors
  - Also odd number of errors
  - And bursts of up to k bits in error

Error Detection in Practice

- CRCs are widely used on links
  - Ethernet, 802.11, ADSL, Cable …
- Checksum used in Internet
  - IP, TCP, UDP … but it is weak
- Parity
  - Is little used