Surfaces of Revolution

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CSE 457
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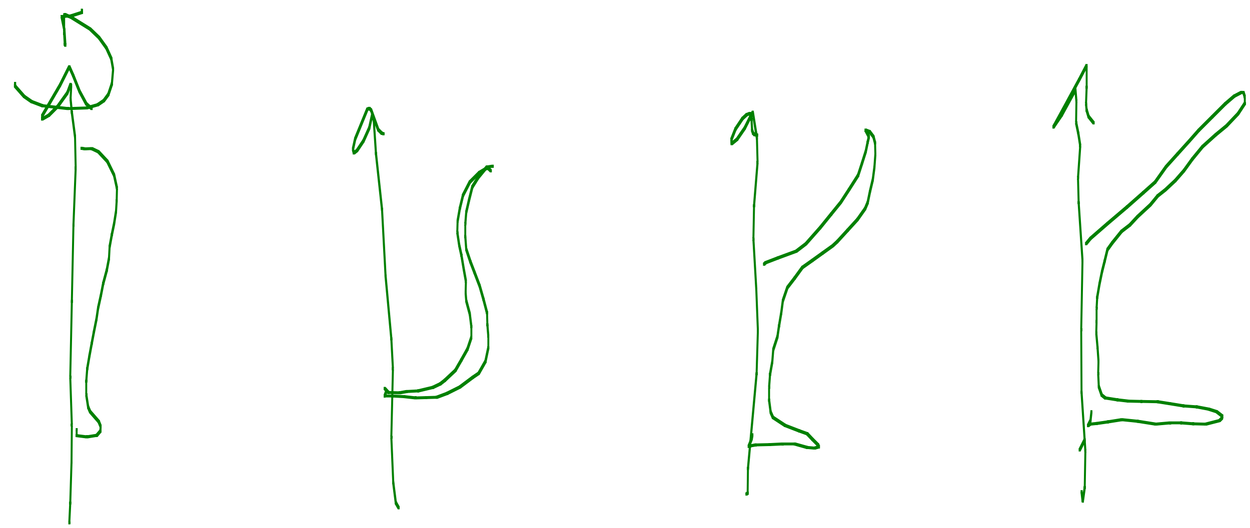
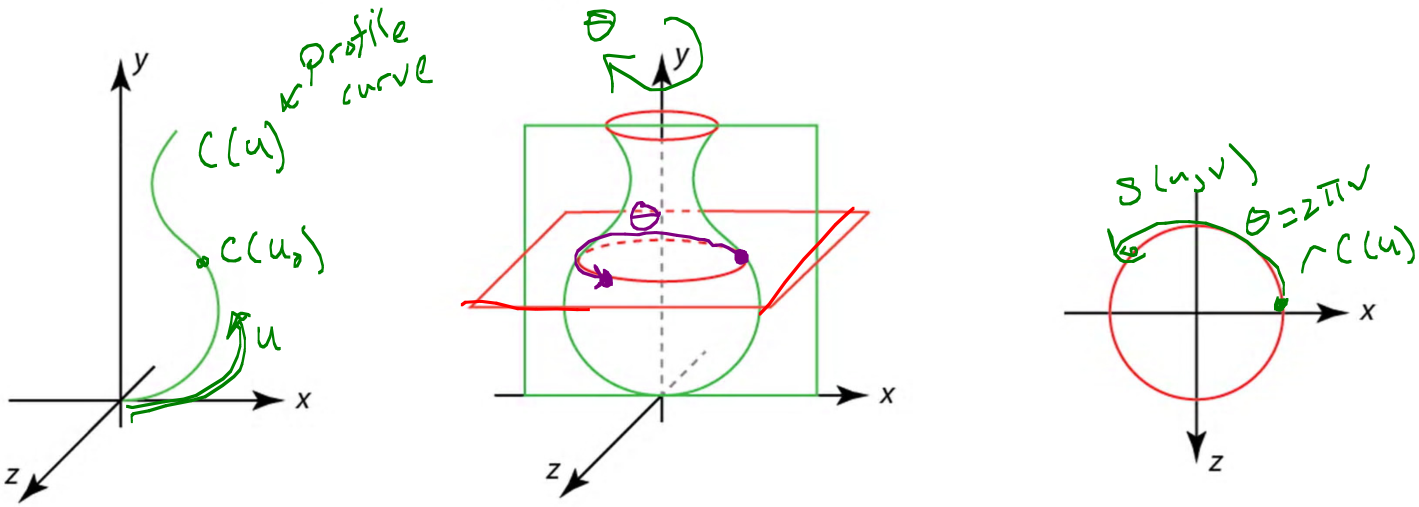
Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Constructing surfaces of revolution



Given: A curve $C(u)$ in the xy-plane:

$$C(u) = \begin{bmatrix} c_x(u) \\ c_y(u) \\ 0 \\ 1 \end{bmatrix}$$

Let $R_y(\theta)$ be a rotation about the y-axis.

Find: A surface $S(u,v)$ which is $C(u)$ rotated about the y-axis, where $u, v \in [0, 1]$.

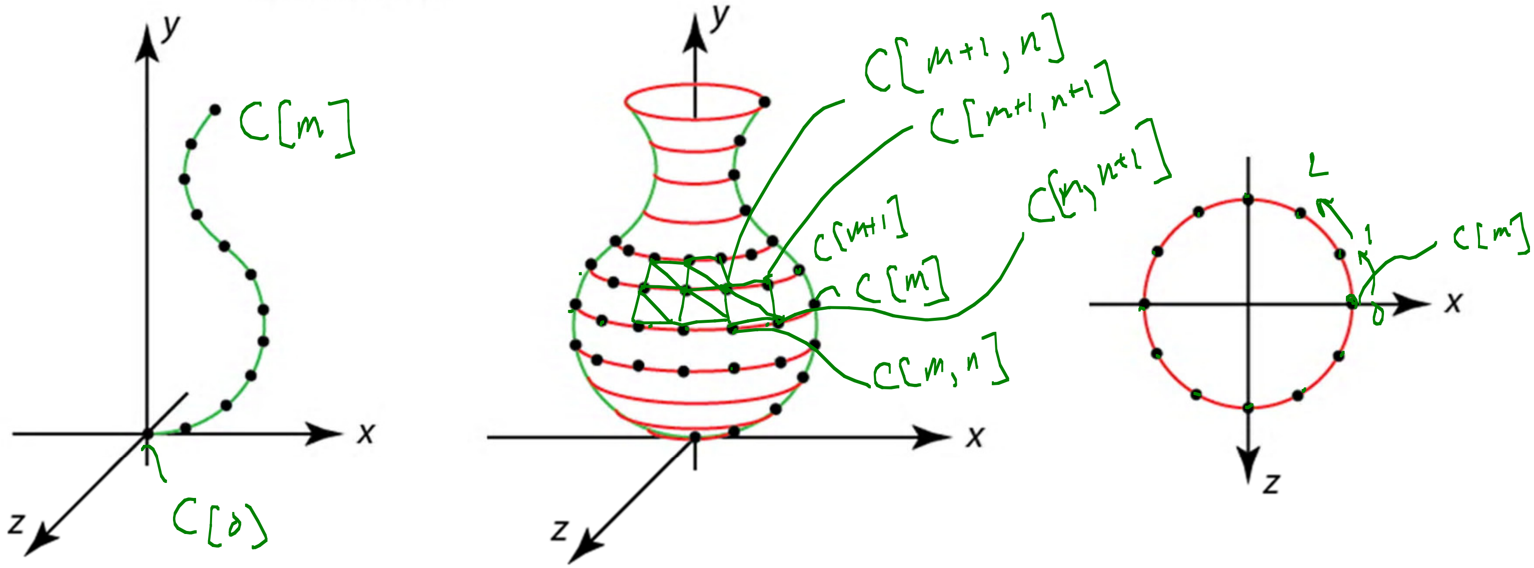
Solution:

$$S(u,\theta) = R_y(\theta)\, C(u)$$

$$S(u,v) = R_y(2\pi v)\, C(u)$$

Constructing surfaces of revolution

We can sample in *u* and *v* to get a grid of points over the surface.



Suppose we sample:

- in *u*, to give $C[m]$ where $m \in [0..M-1]$
- in *v*, to give rotation angle $\theta[n] = 2\pi n/N$ where $n \in [0..N-1]$

We can now write the surface as:

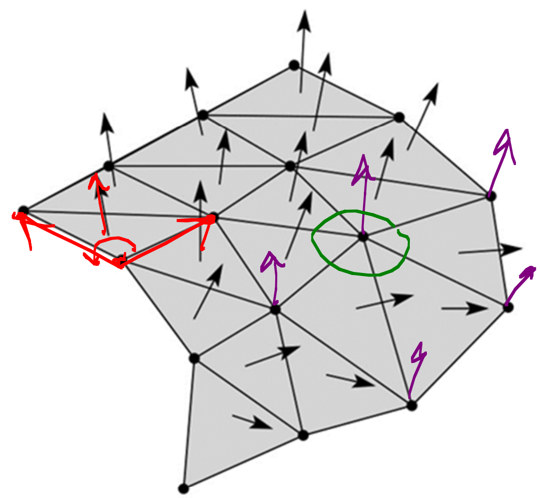
$$S[n,m] = R_y\left(\frac{2\pi n}{N}\right) C[m]$$

How would we turn this into a mesh of triangles?

Surface normals

Now that we describe the surface as a triangle mesh, we need to provide surface normals. As we'll see later, these normals are important for drawing and shading the surface (i.e., for "rendering").

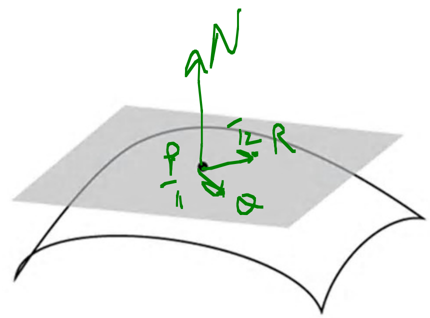
One approach is to compute the normal to each triangle. How do we compute these normals?



Later, we will see that we can get better-looking results by computing the normal at each vertex. How might we do this?

Tangent vectors and tangent planes

$$T \approx \frac{Q-P}{\|Q-P\|}$$

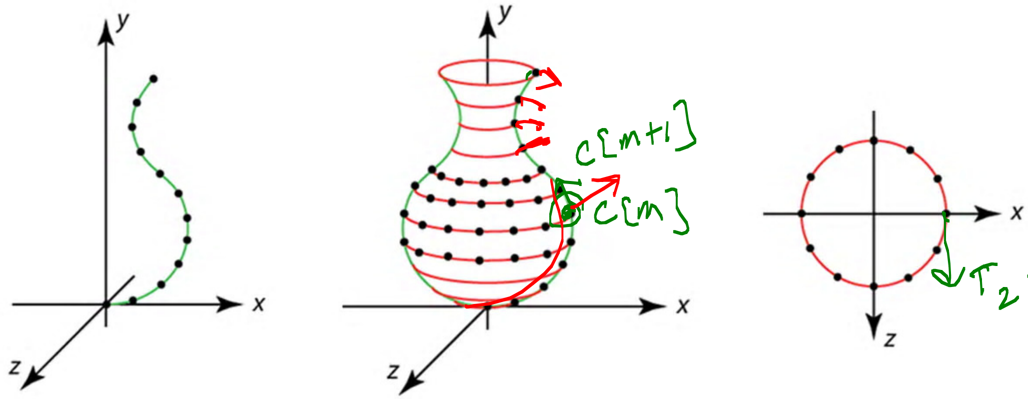


$$T_1 \approx \frac{Q-P}{\|Q-P\|}$$

$$T_2 \approx \frac{R-P}{\|R-Q\|}$$

$$N = \frac{T_1 \times T_2}{\|T_1 \times T_2\|}$$

Normals on a surface of revolution



$$T_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T_1 = C[m+1] - C[m]$$

$$N = T_1 \times T_2$$

$$N[n,m] = R_y\left(\frac{2\pi n}{N}\right) N[0,m]$$

Triangle meshes

How should we generally represent triangle meshes?