Homework #1

Displays, Image Processing,

Affine Transformations, Hierarchical modeling

Assigned: Saturday, October 7th

Due: Friday, October 20th
at the beginning of class

Directions: Please provide short written answers to the following questions, using this page as a cover sheet. Feel free to discuss the problems with classmates, but please ***answer the questions on your own*.**

Name:____

Problem 1: Color LCD displays (16 points)

LCD displays operate on the principle of polarizing light at the entrance to each crystal, twisting the polarization by a voltage controlled amount, then passing the resulting light through a final polarizer. The end result is that the light coming out of the LCD has a particular intensity and (fixed) polarization. The color of the light reaching the viewer is controlled with a color filter (red, green, or blue) over each crystal.

Light coming out of the LCD has "linear polarization," characterized by an angle θ, with a corresponding polarization vector:

$$\mathbf{p} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

A polarizing filter also has an orientation β with polarization vector:

$$\mathbf{f} = \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix}$$

The filter only passes the component of the input polarization aligned with filter's polarization. Specifically, it passes a fraction α of the light:

$$\alpha = \mathbf{f} \bullet \mathbf{p} = \mathbf{f}^T \mathbf{p} = \cos\beta\cos\theta + \sin\beta\sin\theta$$

Now, suppose, through a quirk of manufacturing, all the blue cells for an LCD display have polarization rotated 90 degrees with respect to all the green and red cells. Let's say that the blue polarization is horizontal ($\theta = 0^o$), and the green and red polarizations are vertical ($\theta = 90^o$). Justify your answers to each of the following questions.

- **a)** (4 points) If we display solid white – (R,G,B) = (1,1,1) – and hold against the screen a polarizer in the vertical ($\beta = 90^o$) orientation, what color would we see?
- **b)** (4 points) What color would we see if we then rotated the polarizer to a diagonal ($\beta = 45^o$) orientation?
- **c)** (4 points) What color would we see if we then rotated the polarizer to a horizontal ($\beta = 0^o$) orientation?
- **d)** (4 points) If we keep the filter oriented as in **c**), what color would we see if we now displayed solid green, i.e., (R,G,B) = (0, 1, 0)?

Problem 2: Image processing (30 points)

Suppose you have two digital filters, *a* and *b*, and you want to apply both of them to an image *f*:

$$g[i,j] = a[i,j] * b[i,j] * f[i,j]$$

In fact, convolution is associative, so we can perform this operation in two different orders. One option is to first convolve *b* with *f*, and then convolve *a* with the result:

$$g[i,j] = a[i,j] * (b[i,j] * f[i,j])$$

Another option is to first convolve *a* with *b*, effectively creating a new filter, $a * b$, and then convolve that new filter with *f*:

$$g[i,j] = (a[i,j] * b[i,j]) * f[i,j]$$

The answer you get in either case is identical. (When implemented with finite precision, there will of course be small differences, but we'll neglect this.)

Recall that convolution entails repeated multiplication and addition. For this problem, we'll count a multiplication and addition as one operation called a multiply-accumulate (MAC); in fact, newer processors can perform this operation in one clock cycle. When counting up all the MAC's in a convolution, we'll assume the "accumulator" is initialized to zero and begin adding to it after each multiply. When analyzing the cost of a convolution, you should compute the number of MAC's involved.

Note that for this problem, you can assume that the boundaries of the filters *a* and *b* are implicitly padded with zeroes, but one should of course not count the cost of a MAC involving a product with a zero. For the image *f*, assume that some non-zero extension of the boundary has been defined so that you do you have to count MAC's for portions of a filter that extend outside of the boundary of the image.

You do not need to worry about flipping any of the filters or images in this problem. Justify your answers to the following questions.

a) (4 points) Compute $a * b$ where:

$$a = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \qquad b = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

What kind of filter do you get from this convolution? [Aside: a 2D filter that can be written as the convolution of a 1D horizotnal filter and a 1D vertical filter, as in this sub-problem, is called a "separable" filter.]

b) (4 points) More generally, suppose *a* is a 1D horizontal filter of length *L* and *b* is 1D vertical filter of length *M*. What is the cost of computing $a * b$? How many rows does the resulting filter have? How many columns?

c) (4 points) For the filters in **b**), what is the cost of computing $(a * b) * f$, where *f* is $N \times N$ in size? Let's assume that *N* is much greater than *L* and *M*, and ignore the cost of first computing $a * b$.

Problem 2 (cont'd)

d) (6 points) For the filters in **b**), what is the cost of computing $a*(b*f)$? Assuming L and M are both greater than 1, is this approach more or less efficient than computing $(a*b)*f$ as in **c**)?

e) (6 points) Now suppose a is a 2D filter of size $L \times L$ and b is a 2D filter of size $M \times M$. After computing $a*b$, how many rows and columns will the resulting filter have?

f) (4 points) For the filters in **e**), what is the cost of computing $(a*b)*f$, where f is $N \times N$ in size? Again, let's assume that N is much greater than L and M, and ignore the cost of first computing $a*b$.

g) (6 points) For the filters in **e**), what is the cost of computing $a*(b*f)$? Assuming L and M are both greater than 1, is this approach more or less efficient than computing $(a*b)*f$ as in **f**)?

Problem 3: Rotations (22 points)

Recall that a linear transformation can be determined by mapping the unit Cartesian axes (e.g., the x-, y-, and z-axes in 3D) to new axes in the transformed space. The vectors that define these new axes can simply be entered into the columns of the transformation matrix. Also recall that a rotation maps the set of unit-length, orthogonal vectors pointing along these Cartesian axes into a new set of unit-length, orthogonal vectors.

For each of the sub-problems below, you may assume linear transformation matrices, i.e., no extra bottom row or right column used for affine transformations with translations. Thus, a 3D rotation is represented by a 3x3 matrix, and a 4D rotation is represented by a 4x4 matrix.

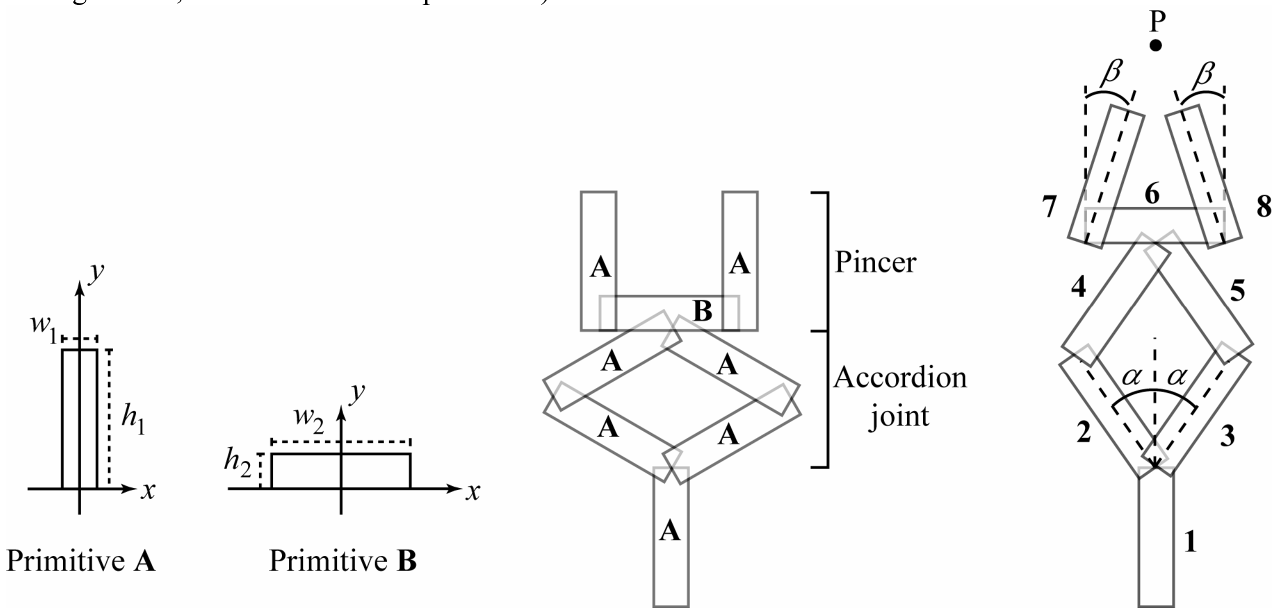
a) (11 points) Given an arbitrary 3D rotation matrix R and its transpose R^T, what does the matrix product $R^T R$ equal? Justify your answer. What does this say about the relationship between R and its transpose?

b) (11 points) Suppose we have a 4D space. We can still define a rotation matrix that has the general properties of rotations stated in the problem introduction. How many degrees of freedom does a 4D rotation matrix have? Justify your answer.

c) (Extra credit: 5 points) How many degrees of freedom does a rotation within an N-dimensional space have?

Problem 4: Hierarchical modeling (32 points)

Suppose you want to model the pincer with accordion joint illustrated below. The model is comprised of 8 parts, using primitives **A** and **B**. The model is shown in two poses below, with the controlling parameters of the model illustrated on the far right. The illustration on the right also shows a point P that the model is reaching toward, as described in sub-problem **c**).



Assume that α and β can take values in the range [0, 90°]. Also assume that all parts use primitive **A**, except for part **6**, which uses primitive **B**. The model on the left shows the primitives used, the model on the right shows the enumeration (naming) of the parts.

The following transformations are available to you:

- R(θ) – rotate by θ degrees (counter clockwise)
- T(a, b) – translate by $\begin{bmatrix} a \\ b \end{bmatrix}$

a) (20 points) Construct a tree to describe this hierarchical model using part **1** as the root. Along each of the edges of the tree, write expressions for the transformations that are applied along that edge, using the notation given above (you do not need to write out the matrices). Remember that the order of transformations is important! Show your work wherever the transformations are not "obvious."

b) (4 points) Write out the full transformation expression for part **7**.

c) (8 points) Suppose the primitives are infinitesimally thin, $w_1 = h_2 = 0$, and have lengths $h_1 = 10$ and $w_2 = 12$. Assume that part **1** sits right on the origin in world coordinates. What would the α and β parameters have to be so that the model extends out and closes the pincer just enough to precisely grasp the point $P = \begin{bmatrix} 0 & 28 \end{bmatrix}^T$, in world coordinates. Show your work.