Topics in Articulated Animation

Reading

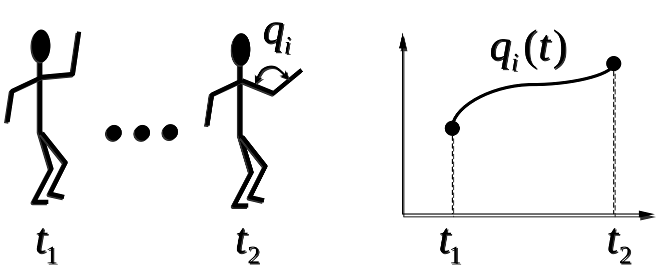
Shoemake, “Quaternions Tutorial”

Animation

Articulated models:

- rigid parts
- connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.



Character Representation

Character Models are rich, complex

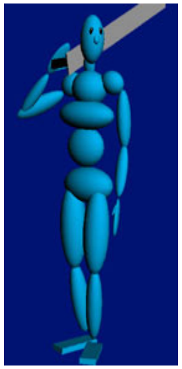
- hair, clothes (particle systems)
- muscles, skin (FFD’s *etc.*)

Focus is rigid-body Degrees of Freedom (DOFs)

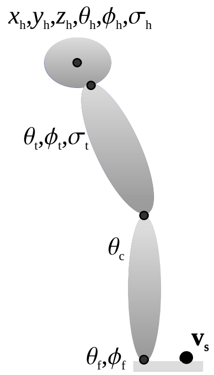
- joint angles

Simple Rigid Body → Skeleton

vs.

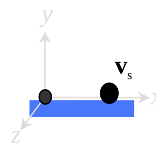


Computing a Sensor Position



Forward kinematics

- uses vector-matrix multiplication
- transformation matrix is composition of all joint transforms between sensor/effector and root



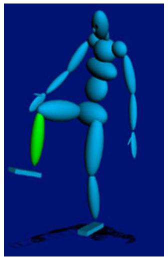
$$\mathbf{v}_w = \mathbf{T}(x_h, y_h, z_h)\mathbf{R}(\theta_h, \phi_h, \sigma_h)\ \mathbf{TR}(\theta_t, \phi_t, \sigma_t)\ \mathbf{TR}(\theta_c)\ \mathbf{TR}(\theta_f, \phi_f)\ \mathbf{v}_s$$

Joints = Rotations

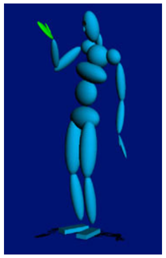
To specify a pose, we specify the joint-angle rotations

Each joint can have up to three rotational DOFs

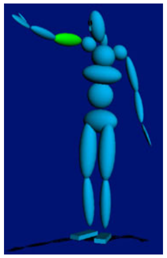
1 DOF: knee



2 DOF: wrist



3 DOF: arm



Euler angles

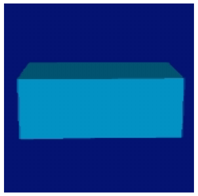
An Euler angle is a rotation about a single Cartesian axis

Create multi-DOF rotations by concatenating Eulers

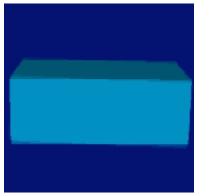
Can get three DOF by concatenating:

Euler-X

Euler-Y



Euler-Z



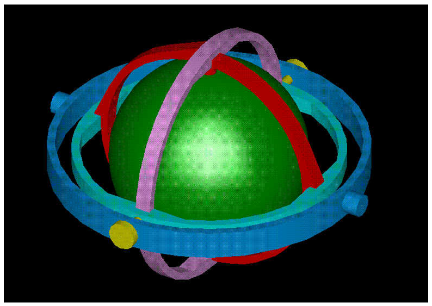
Singularities

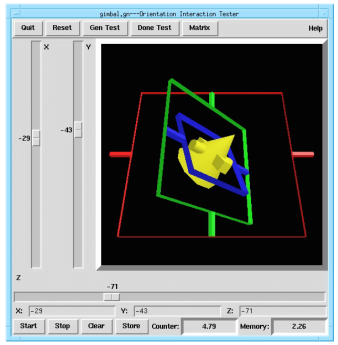
What *is* a singularity?

- continuous subspace of parameter space all of whose elements map to same rotation

Why is this bad?

- induces **gimbal lock** - two or more axes align, results in loss of rotational DOFs (*i.e.* derivatives)





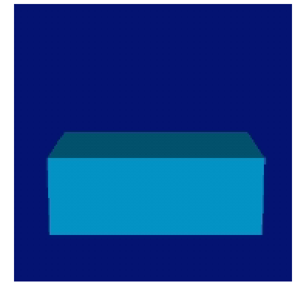
Singularities in Action

An object whose orientation is controlled by Euler rotation XYZ(θ,ϕ,σ)

$$\mathbf{R}(\theta,\phi,\sigma) = \mathbf{R}_x(\theta)\cdot\mathbf{R}_y(\phi)\cdot\mathbf{R}_z(\sigma)$$

R(0,0,0) : Okay

R(0, ± 90^o, 0) : X and Z axes align

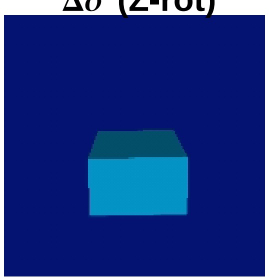


Eliminates a DOF

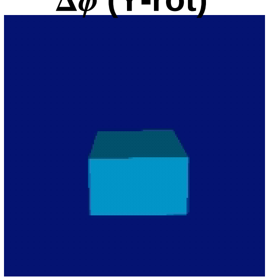
In this configuration, changing θ (X Euler angle) and σ (Z Euler angle) produce the same result.

No way to rotate around world X axis!

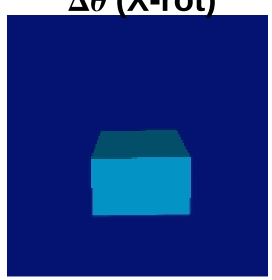
$\Delta\sigma$ (Z-rot)



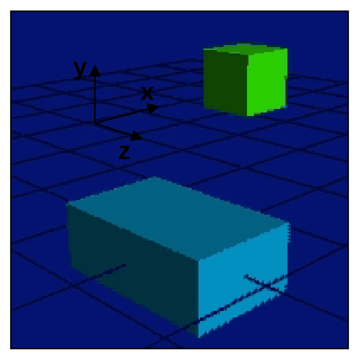
$\Delta\phi$ (Y-rot)



$\Delta\theta$ (X-rot)



Resulting Behavior



No applied force or other stimuli can induce rotation about world X-axis

The object locks up!!

Singularities in Euler Angles

Cannot be avoided (occur at 0° or 90°)

Difficult to work around

But, only affects three DOF rotations

Other Properties of Euler Angles

Several important tasks are easy:

- interactive specification (sliders, *etc.*)
- joint limits
- Euclidean interpolation (Hermites, Beziers, *etc.*)
  - May be funky for tumbling bodies
  - fine for most joints

Quaternions

But… singularities are unacceptable for IK, optimization

Traditional solution: Use unit quaternions to represent rotations

- S^3 has same topology as rotation space (a sphere), so no singularities

History of Quaternions

Invented by Sir William Rowan Hamilton in 1843

$$H = w + \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$$
$$\text{where } \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

I still must assert that this discovery appears to me to be as important for the middle of the nineteenth century as the discovery of fluxions [the calculus] was for the close of the seventeenth.

Hamilton

[quaternions] … although beautifully ingenious, have been an unmixed evil to those who have touched them in any way.

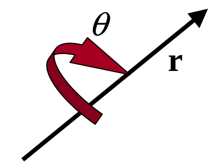
Thompson

Quaternion as a 4 vector

$$\mathbf{q} = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} w \\ \mathbf{v} \end{pmatrix}$$

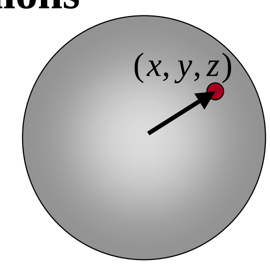
Axis-angle rotation as a quaternion

$$\mathbf{q} = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} w \\ \mathbf{v} \end{pmatrix}$$



$$\mathbf{q} = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\mathbf{r} \end{pmatrix}$$

Unit Quaternions

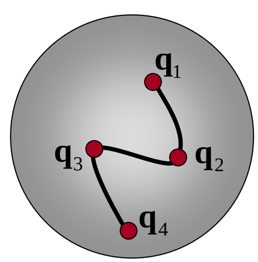


$$\mathbf{q} = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}$$

$$w = \sqrt{1-(x^2+y^2+z^2)}$$

$$|\mathbf{q}| = 1$$

$$x^2+y^2+z^2+w^2=1$$



Quaternion Product

$$\begin{pmatrix} w_1 \\ \mathbf{v}_1 \end{pmatrix}\begin{pmatrix} w_2 \\ \mathbf{v}_2 \end{pmatrix} = \begin{pmatrix} w_1 w_2 - \mathbf{v}_1 \cdot \mathbf{v}_2 \\ w_1\mathbf{v}_2 + w_2\mathbf{v}_1 + \mathbf{v}_1 \times \mathbf{v}_2 \end{pmatrix}$$

$$\begin{pmatrix} w_1 \\ \mathbf{v}_1 \end{pmatrix}\begin{pmatrix} w_2 \\ \mathbf{v}_2 \end{pmatrix} \neq \begin{pmatrix} w_2 \\ \mathbf{v}_2 \end{pmatrix}\begin{pmatrix} w_1 \\ \mathbf{v}_1 \end{pmatrix}$$

Quaternion Conjugate

$$\mathbf{q}^* = \begin{pmatrix} w_1 \\ \mathbf{v}_1 \end{pmatrix}^* = \begin{pmatrix} w_1 \\ -\mathbf{v}_1 \end{pmatrix}$$

$$(\mathbf{p}^*)^* = \mathbf{p}$$

$$(\mathbf{pq})^* = \mathbf{q}^*\mathbf{p}^*$$

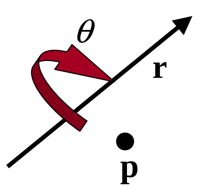
$$(\mathbf{p}+\mathbf{q})^* = \mathbf{p}^* + \mathbf{q}^*$$

Quaternion Inverse

$$\mathbf{q}^{-1}\mathbf{q} = 1$$

$$\mathbf{q}^{-1} = \mathbf{q}^* / |\mathbf{q}| = \begin{pmatrix} w \\ -\mathbf{v} \end{pmatrix} / |\mathbf{q}| = \begin{pmatrix} w \\ -\mathbf{v} \end{pmatrix} / (w^2 + \mathbf{v}\cdot\mathbf{v})$$

Quaternion Rotation

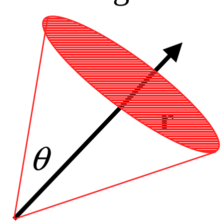


$$\begin{aligned}\mathbf{qpq}^{-1} &= \begin{pmatrix} w \\ \mathbf{v} \end{pmatrix}\begin{pmatrix} 0 \\ \mathbf{p} \end{pmatrix}\begin{pmatrix} w \\ -\mathbf{v} \end{pmatrix} \\ &= \begin{pmatrix} w \\ \mathbf{v} \end{pmatrix}\begin{pmatrix} \mathbf{p}\cdot\mathbf{v} \\ w\mathbf{p} - \mathbf{p}\times\mathbf{v} \end{pmatrix} \\ &= \begin{pmatrix} w\mathbf{p}\cdot\mathbf{v} - w\mathbf{p}\cdot\mathbf{v} = 0 \\ w(w\mathbf{p} - \mathbf{pv}) + (\mathbf{p}\cdot\mathbf{v})\mathbf{v} + \mathbf{v}(w\mathbf{p} - \mathbf{p}\times\mathbf{v}) \end{pmatrix}\end{aligned}$$

What about a quaternion product $\mathbf{q}_1\mathbf{q}_2$?

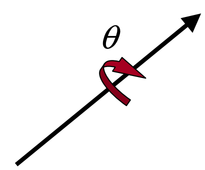
Quaternion constraints

Restricting the rotation cone



$$\frac{1-\cos(\theta_x)}{2} = q_y^{\,2} + q_z^{\,2}$$

Restricting the rotation twist around an axis



$$\tan(\theta/2) = \frac{q_{axis}}{q_w}$$

Matrix Form

$$\mathbf{q} = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} 1-2y^2-2z^2 & 2xy+2wz & 2xz-2wy \\ 2xy-2wz & 1-2x^2-2z^2 & 2yz+2wx \\ 2xz+2wy & 2yz-2wx & 1-2x^2-2y^2 \end{pmatrix}$$

Quaternions: What Works

Simple formulae for converting to rotation matrix

Continuous derivatives - no singularities

“Optimal” interpolation - geodesics map to shortest paths in rotation space

Nice calculus (corresponds to rotations)

What Hierarchies Can and Can’t Do

Advantages:

- Reasonable control knobs
- Maintains structural constraints

Disadvantages:

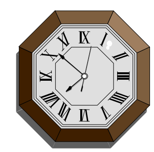
- Doesn’t always give the “right” control knobs
  - e.g. hand or foot position - re-rooting may help
- Can’t do closed kinematic chains (keep hand on hip)
- Other constraints: do not walk through walls

Procedural Animation

Transformation parameters as functions of other variables

Simple example:

- a clock with second, minute and hour hands
- hands should rotate together
- express all the motions in terms of a “seconds” variable
- whole clock is animated by varying the seconds parameter



Models as Code: draw-a-bug

```
void draw_bug(walk_phase_angle, xpos, ypos zpos){
  pushmatrix
  translate(xpos,ypos,zpos)
  calculate all six sets of leg angles based on
    walk phase angle.
  draw bug body
  for each leg:
    pushmatrix
    translate(leg pos relative to body)
    draw_bug_leg(theta1&theta2 for that leg)
    popmatrix
  popmatrix
}

void draw_bug_leg(float theta1, float theta2){
  glPushMatrix();
  glRotatef(theta1,0,0,1);
  draw_leg_segment(SEGMENT1_LENGTH)
  glTranslatef(SEGMENT1_LENGTH,0,0);
  glRotatef(theta2,0,0,1);
  draw_leg_segment(SEGMENT2_LENGTH)
  glPopMatrix();
}
```


Motion Editing

Tools for transforming already existing animations

Hard Example

In the figure below, what expression would you use to calculate the arm's rotation angle to keep the tip on the star-shaped wheel as the wheel rotates???