

## Subdivision curves and surfaces

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## Reading

Recommended:

- ♦ Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 6.1-6.3, 10.2, A.5.

Note: there is an error in Stollnitz, et al., section A.5. Equation A.3 should read:

$$MV = V\Lambda$$

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## Subdivision curves

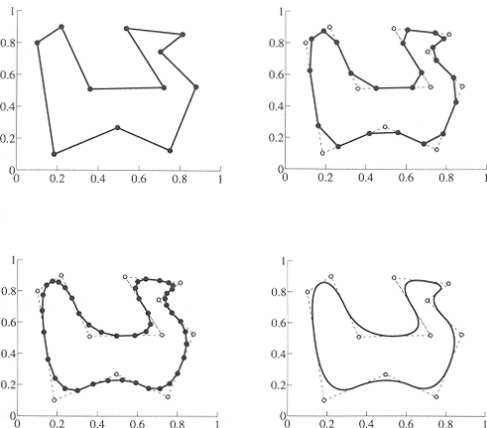
Idea:

- ♦ repeatedly refine the control polygon

$$P^1 \rightarrow P^2 \rightarrow P^3 \rightarrow \dots$$

- ♦ curve is the limit of an infinite process

$$Q = \lim_{j \rightarrow \infty} P^j$$

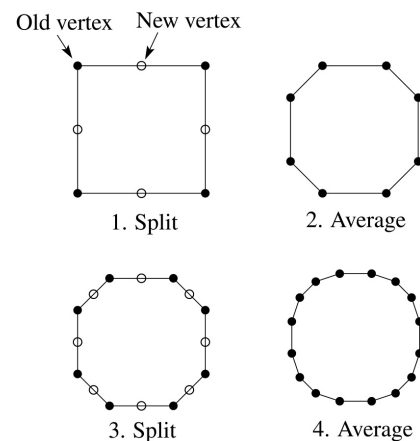


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## Chaikin's algorithm

Chaikin introduced the following "corner-cutting" scheme in 1974:

- ♦ Start with a piecewise linear curve
- ♦ Insert new vertices at the midpoints (the **splitting step**)
- ♦ Average each vertex with the "next" (clockwise) neighbor (the **averaging step**)
- ♦ Go to the splitting step



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## Averaging masks

The limit curve is a quadratic B-spline!

Instead of averaging with the nearest neighbor, we can generalize by applying an **averaging mask** during the averaging step:

$$r = (\dots, r_{-1}, r_0, r_1, \dots)$$

In the case of Chaikin's algorithm:

$$r =$$

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## Lane-Riesenfeld algorithm (1980)

Use averaging masks from Pascal's triangle:

$$r = \frac{1}{2^n} \left( \binom{n}{0}, \binom{n}{1}, \dots, \binom{n}{n} \right)$$

Gives B-splines of degree  $n+1$ .

$n=0$ :

$n=1$ :

$n=2$ :

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## Subdivide ad nauseum?

After each split-average step, we are closer to the **limit curve**.

How many steps until we reach the final (limit) position?

Can we push a vertex to its limit position without infinite subdivision? Yes!

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## Recipe for subdivision curves

After subdividing and averaging a few times, we can push each vertex to its limit position by applying an **evaluation mask**.

Each subdivision scheme has its own evaluation mask, mathematically determined by analyzing the subdivision and averaging rules.

For Lane-Riesenfeld cubic B-spline subdivision, we get:

$$\frac{1}{6} (1 \ 4 \ 1)$$

Now we can cook up a simple procedure for creating subdivision curves:

- ◆ Subdivide (split+average) the control polygon a few times. Use the averaging mask.
- ◆ Push the resulting points to the limit positions. Use the evaluation mask.

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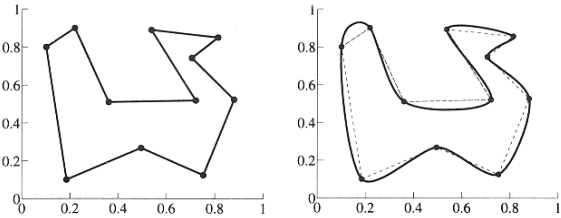
## DLG interpolating scheme (1987)

Slight modification to subdivision algorithm:

- ♦ splitting step introduces midpoints
- ♦ averaging step *only changes midpoints*

For DLG (Dyn-Levin-Gregory), use:

$$r = \frac{1}{16}(-2, 5, 10, 5, -2)$$

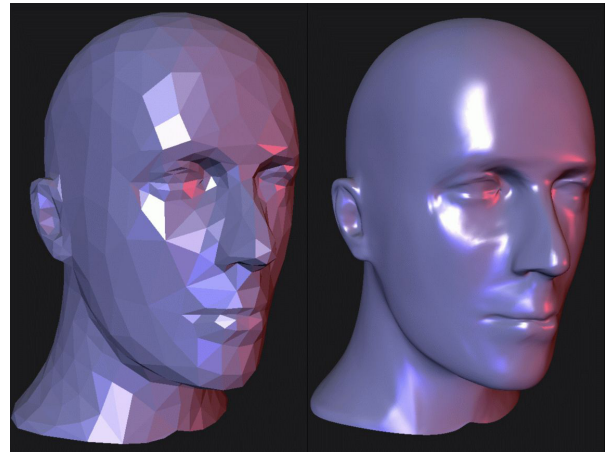


Since we are only changing the midpoints, the points after the averaging step do not move.

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## Building complex models

We can extend the idea of subdivision from curves to surfaces...



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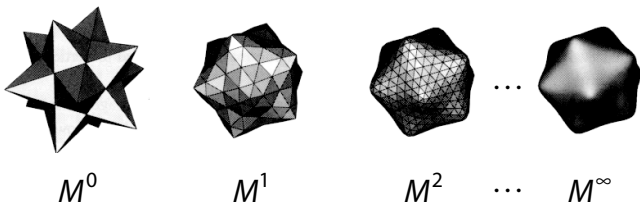
## Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$\sigma = \lim_{j \rightarrow \infty} M^j$$

using splitting and averaging steps.

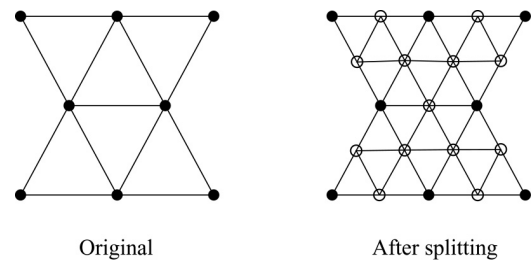


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## Triangular subdivision

There are a variety of ways to subdivide a polygon mesh.

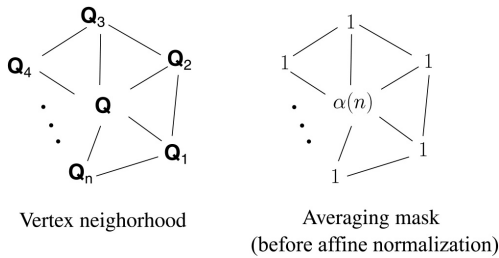
A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:



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## Loop averaging step

Once again we can use **masks** for the averaging step:



$$\mathbf{Q} \leftarrow \frac{\alpha(n)\mathbf{Q} + \mathbf{Q}_1 + \dots + \mathbf{Q}_n}{\alpha(n) + n}$$

where

$$\alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3+2\cos(2\pi/n))^2}{32}$$

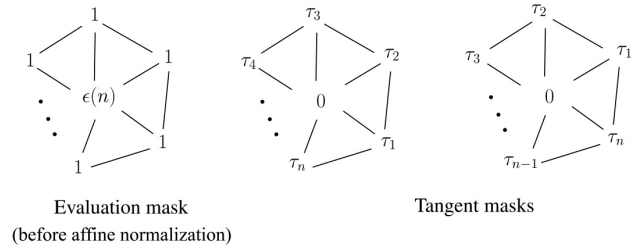
These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as  $G^1$  continuity for surfaces.

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## Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



$$\mathbf{Q}^\infty = \frac{\epsilon(n)\mathbf{Q} + \mathbf{Q}_1 + \dots + \mathbf{Q}_n}{\epsilon(n) + n}$$

$$\mathbf{T}_1^\infty = \tau_1(n)\mathbf{Q}_1 + \tau_2(n)\mathbf{Q}_2 + \dots + \tau_n(n)\mathbf{Q}_n$$

$$\mathbf{T}_2^\infty = \tau_n(n)\mathbf{Q}_1 + \tau_1(n)\mathbf{Q}_2 + \dots + \tau_{n-1}(n)\mathbf{Q}_n$$

where

$$\epsilon(n) = \frac{3n}{\beta(n)} \quad \tau_i(n) = \cos(2\pi i/n)$$

How do we compute the normal?

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## Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

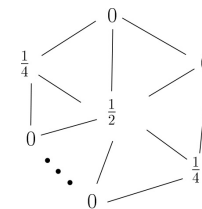
- ◆ Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- ◆ Compute two tangent vectors using the tangent masks.
- ◆ Compute the normal from the tangent vectors.
- ◆ Push the resulting points to the limit positions. Use the evaluation mask.
- ◆ Render!

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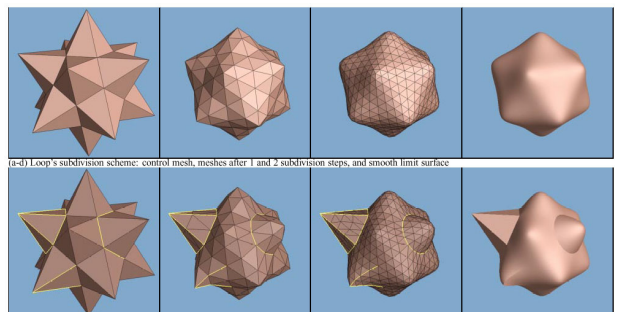
## Adding creases without trim curves

In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision mask:



This gives rise to  $G^0$  continuous surfaces (i.e., having positional but not tangent plane continuity)



(a-d) Loop's subdivision scheme: control mesh, meshes after 1 and 2 subdivision steps, and smooth limit surface

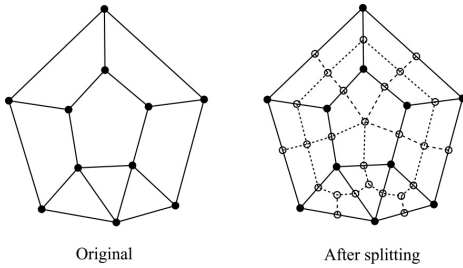
(e-h) Our piecewise smooth subdivision scheme: tagged control mesh, meshes after 1 and 2 subdivision steps, and piecewise smooth limit surface

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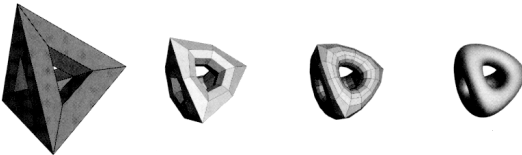
## Catmull-Clark subdivision

4:1 subdivision of triangles is sometimes called a **face scheme** for subdivision, as each face begets more faces.

An alternative face scheme starts with arbitrary polygon meshes and inserts vertices along edges and at face centroids:



### Catmull-Clark subdivision:



Note: after the first subdivision, all polygons are quadrilaterals in this scheme.

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## Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):



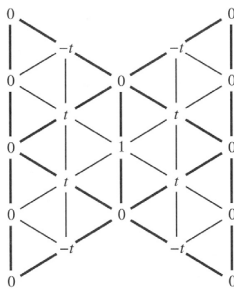
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## Interpolating subdivision surfaces

Interpolating schemes are defined by

- ♦ splitting
- ♦ averaging only new vertices

The following averaging mask is used in **butterfly subdivision**:



Setting  $t=0$  gives the original polyhedron, and increasing small values of  $t$  makes the surface smoother, until  $t=1/8$  when the surface is provably  $G^1$ .

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## Summary

What to take home:

- ♦ The meanings of all the **boldfaced** terms.
- ♦ How to perform the splitting and averaging steps on subdivision curves.
- ♦ How to perform mesh splitting steps for subdivision surfaces, especially Loop.
- ♦ How to construct and render subdivision surfaces from their averaging masks, evaluation masks, and tangent masks.

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