Subdivision surfaces

Reading

Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 10.2.

Subdivision curves

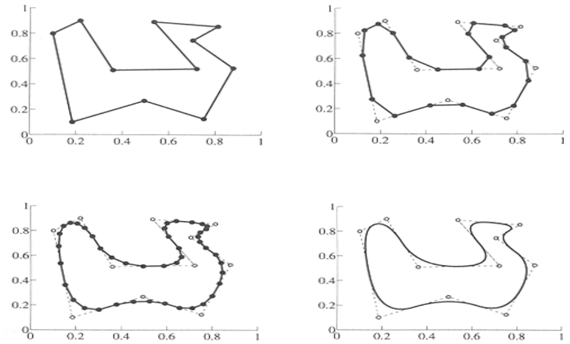
Idea:

- repeatedly refine the control polygon

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow \cdots$$

$$C = \lim_{i \to \infty} P_i$$

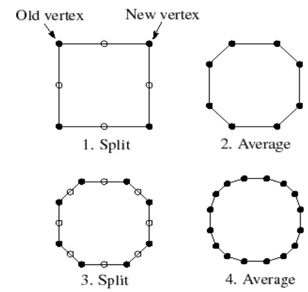
- curve is the limit of an infinite process



Chaikin's algorithm

Chakin introduced the following "corner-cutting" scheme in 1974:

- Start with a piecewise linear curve
- Insert new vertices at the midpoints (the **splitting step**)
- Average each vertex with the "next" neighbor (the **averaging step**)
- Go to the splitting step



Averaging masks

The limit curve is a quadratic B-spline!

Instead of averaging with the nearest neighbor, we can generalize by applying an **averaging mask** during the averaging step:

$$r = (\ldots, r_{-1}, r_0, r_1, \ldots)$$

In the case of Chaikin's algorithm:

$$r =$$

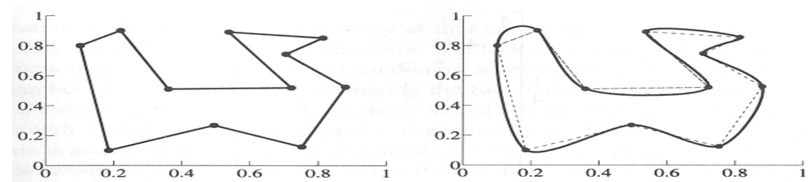
DLG interpolating scheme (1987)

Slight modification to algorithm:

- splitting step introduces midpoints
- averaging step *only changes midpoints*

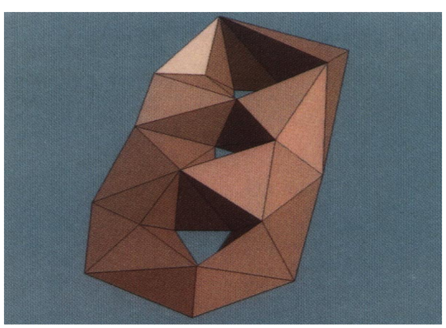
For DLG (Dyn-Levin-Gregory), use:

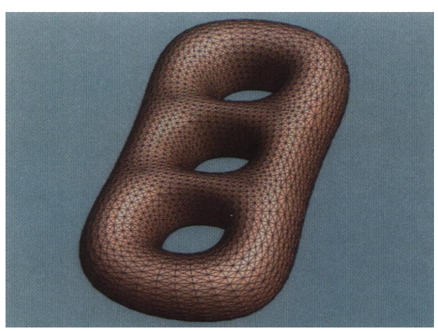
$$r = \frac{1}{16}(-2, 6, 10, 6, -2)$$



Since we are only changing the midpoints, the points after the averaging step do not move.

Building complex models





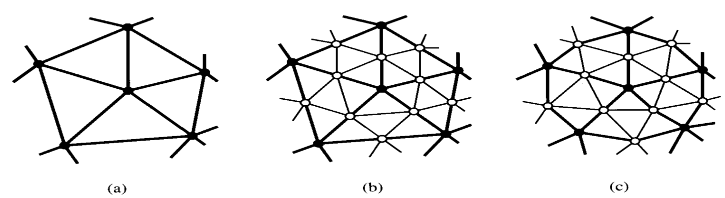
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$\sigma = \lim_{j \to \infty} M^j$$

using splitting and averaging steps.

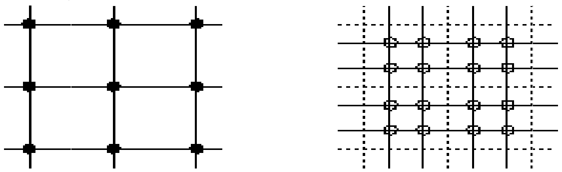


There are two types of splitting steps:

- **vertex schemes**
- **face schemes**

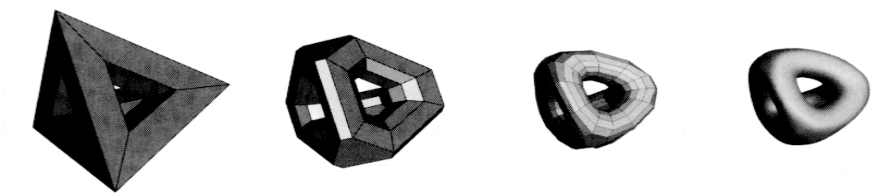
Vertex schemes

A vertex surrounded by n faces is split into n subvertices, one for each face:



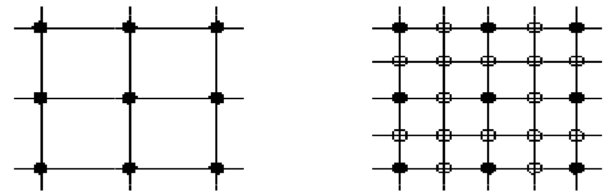
Original   After splitting

Doo-Sabin subdivision:



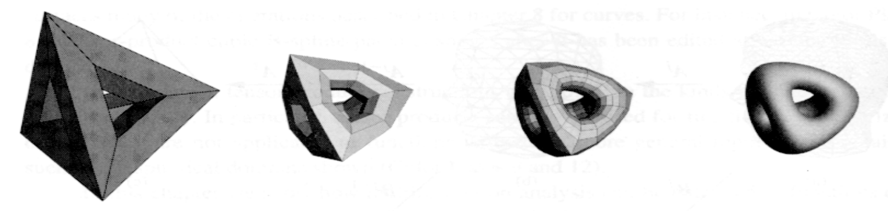
Face schemes

Each quadrilateral face is split into four subfaces:



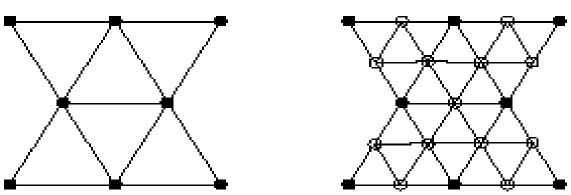
Original   After splitting

Catmull-Clark subdivision:



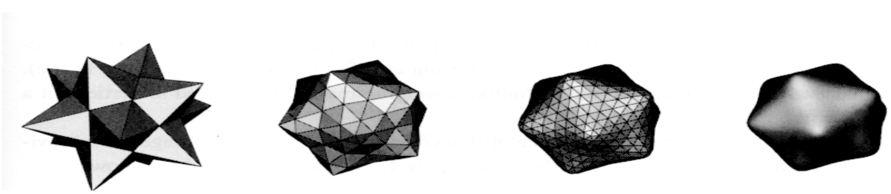
Face schemes, cont.

Each triangular face is split into four subfaces:



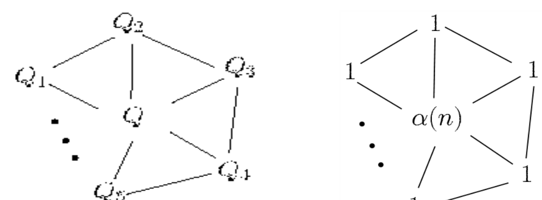
Original   After splitting

Loop subdivision:



Averaging step

Once again we can use **masks** for the averaging step:



Vertex labeling   Averaging mask

$$Q \leftarrow \frac{\alpha(n) + Q_1 + \cdots + Q_n}{\alpha(n) + n}$$

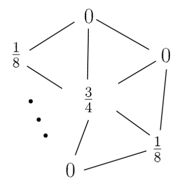
where

$$\alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \qquad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi / n))^2}{32}$$

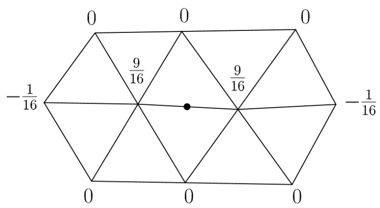
(carefully chosen to ensure smoothness.)

Adding creases without trim curves

Sometimes, particular feature such as a crease should be preserved. With NURBS surfaces, this required the use of trim curves.
For subdivision surfaces, we just modify the subdivision mask:

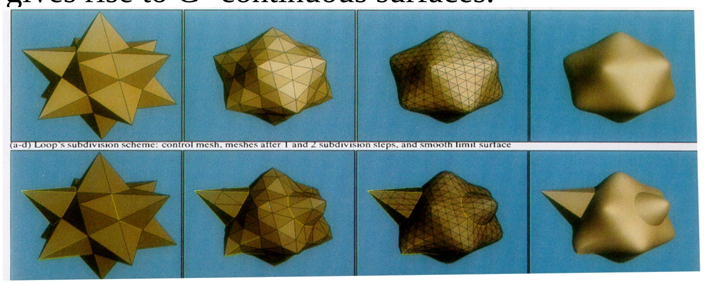


Loop crease/boundary edge



Buttery crease/boundary edge

This gives rise to G^0 continuous surfaces.



Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces of the kind found in Geri's Game: