19. Subdivision surfaces

Reading

Recommended:

- Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications,* 1996, section 10.2.

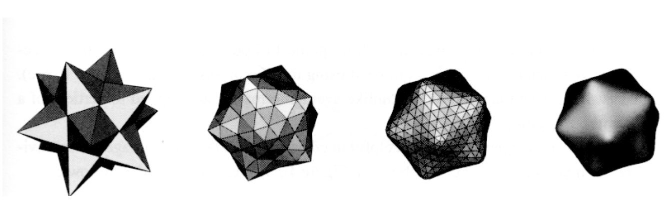
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$\sigma = \lim_{j\to\infty} M^j$$

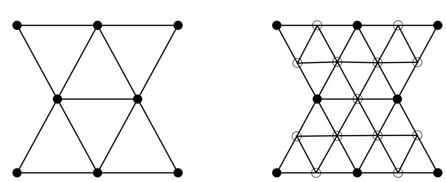
using splitting and averaging steps.



Triangular subdivision

There are a variety of ways to subdivide a poylgon mesh.

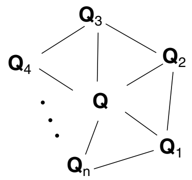
A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:



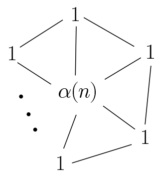
Original After splitting

Loop averaging step

Once again we can use **masks** for the averaging step:



Vertex neighorhood



Averaging mask

$$\mathbf{Q} \leftarrow \frac{\alpha(n)\mathbf{Q} + \mathbf{Q}_1 + \cdots + \mathbf{Q}_n}{\alpha(n) + n}$$

where

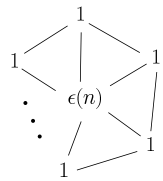
$$\alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \qquad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi/n))^2}{32}$$

These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

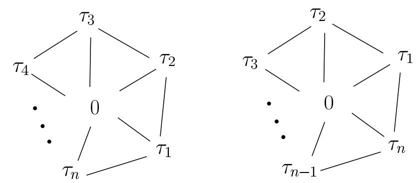
Note: tangent plane continuity is also know as G^1 continuity.

Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



Evaluation mask



Tangent masks

$$\mathbf{Q}^{\infty} = \frac{\varepsilon(n)\mathbf{Q} + \mathbf{Q}_1 + \cdots + \mathbf{Q}_n}{\varepsilon(n) + n}$$

$$\mathbf{T}_1^{\infty} = \tau_1(n)\mathbf{Q}_1 + \tau_2(n)\mathbf{Q}_2 + \cdots + \tau_n(n)\mathbf{Q}_n$$

$$\mathbf{T}_2^{\infty} = \tau_n(n)\mathbf{Q}_1 + \tau_1(n)\mathbf{Q}_2 + \cdots + \tau_{n-1}(n)\mathbf{Q}_n$$

where

$$\varepsilon(n) = \frac{3n}{\beta(n)} \qquad \tau_i(n) = \cos(2\pi i/n)$$

How do we compute the normal?

Recipe for subdivision surfaces

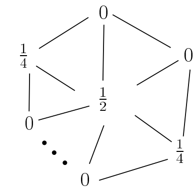
As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!

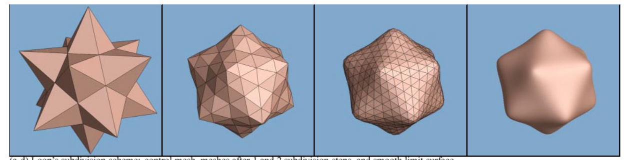
Adding creases without trim curves

In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

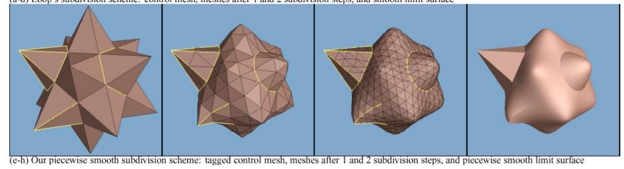
For subdivision surfaces, we can just modify the subdivision mask:



This gives rise to G^0 continuous surfaces (i.e., having positional but not tangent plane discontinuity)



(a-d) Loop's subdivision scheme: control mesh, meshes after 1 and 2 subdivision steps, and smooth limit surface



(e-h) Our piecewise smooth subdivision scheme: tagged control mesh, meshes after 1 and 2 subdivision steps, and piecewise smooth limit surface

Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

Summary

What to take home:

- The meanings of all the **boldfaced** terms.
- How to construct and render Loop subdivision surfaces from the averaging masks, evaluation masks, and tangent masks.