Name: ______________________

1	/15
2	/ 9
3	/13
4	/13
Total	/50

CSE 431: Introduction to the Theory of Computation
Midterm Exam

May 1, 2000

Instructions:

- Give your answers in the spaces provided on these sheets.
- No books, no notes permitted.
- There are 4 problems on this exam. You will have 50 minutes in total. Don't spend too much time on any one problem.

1. (15 points) Suppose that A and B are languages satisfying $A \leq_m B$. For each of the following, state whether it is true or false. No justification is needed.

   (a) If A is undecidable, then B is undecidable.

   (b) If B is undecidable, then A is undecidable.

   (c) If A is Turing-recognizable, then B is Turing-recognizable.

   (d) If B is Turing-recognizable, then A is Turing-recognizable.

   (e) If $\overline{B}$ is Turing-recognizable, then $\overline{A}$ is Turing-recognizable.

2. (9 points) What is wrong with the following proof that A_{TM} is undecidable? "Suppose by way of contradiction that A_{TM} is decided by some Turing machine D. Given input $< M, w >$, D can simulate M on input w, and accept if M accepts w or reject if M rejects w. But if M loops on input w, D will also loop while simulating M. This contradicts the assumption that D is a decider. Therefore the assumption that D exists is false, and A_{TM} is undecidable."

3. (13 points) Suppose that a Turing machine is given a C++ program $< P >$ as input, where P is a program that takes no input. Prove that it is undecidable to determine whether P halts or goes into an infinite loop. (This has obvious unfortunate consequences for compiler design.)

   (Hint: Reduce A_{TM} to this problem. You may find it convenient to represent the Turing machine tape by a doubly-linked list, so that you don't have to worry about how long the tape is. You may ignore the possibility that the C++ program runs out of memory.)

4. (13 points) Let

$$NQ = \{< M, w >| \text{ NTM } M \text{ accepts } w \text{ on some computation branch within } |w|^2 \text{ steps}\}.$$

Notice that it is possible that some computation branches of M on input w do not halt.

Is NQ decidable? Give a proof to support your answer.