CSE 431: Theory of Computation
Assignment #6
May 17, 2000
due: Wednesday, May 24

In all problems, you should assume that integers are given in binary, vertices in a graph $G = (V, E)$ are listed in the input as the binary numbers from 1 to $|V|$, and E is given as a list of pairs of such binary numbers.

1. Problem 7.16. A path is *simple* if and only if it doesn't repeat any vertex. Note that SPATH is the language that I called USP on previous assignments.

2. Problem 7.21.

3. Problem 7.22. (Hint: somewhere in part (b) it will be helpful to use part (a) to ensure that the variable b is assigned the value False.)

4. Problem 7.23. Warning: the k in the definition of the language $MAX-CUT$ is not the same k used later in the problem, the number of clauses. To avoid confusion, change k to k' in the definition of the language.

   Continuation of the book's hint: Suppose that you have a cut in which the $3k$ nodes labeled x are split between S and T, say i of them in S and $3k-i$ of them in T, where $1 \leq i \leq 3k/2$. Then I claim that at least $3k$ of the edges connecting nodes labeled x and nodes labeled $\overline{x}$ are not in the cut. Here is the reason: suppose j nodes labeled $\overline{x}$ are in T and the remaining $3k-j$ are in S. Then the number of $x-\overline{x}$ edges not in the cut is

$$\begin{aligned} i(3k-j) + j(3k-i) &= 3ki + 3kj - 2ij \\ &\geq 3k \cdot 1 + 3kj - 2j(3k/2) \\ &= 3k. \end{aligned}$$