CSE 431: Theory of Computation
Assignment #5
May 8, 2000
due: Monday, May 15

In all problems, you should assume that integers are given in binary, vertices in a graph $G = (V, E)$ are listed in the input as the binary numbers from 1 to $|V|$, and E is given as a list of pairs of such binary numbers.

1. Suppose that B is NL-complete. Prove that $B \in L$ if and only if $L = NL$. Corollary 8.19 in the textbook is one direction of this result. Of course you may use Theorem 8.18 freely in your proof.

2. (a) Prove that $\leq_L$ is transitive. Be sure that your Turing machine only uses $O(\log n)$ worktape space for all intermediate results.

   (b) Suppose that A is NL-complete, $B \in NL$, and $A \leq_L B$. Prove that B is NL-complete.

3. Let

$$USP = \{< G, s, t, k > | \ G \text{ is an undirected graph with a path of length at most } k \text{ between vertices } s \text{ and } t\}.$$

   Prove that USP is NL-complete. (Hint: given Problem 2 from Assignment 4 and Problem 2 above, it suffices to prove that $PATH \leq_L USP$. If $G = (V, E)$ is the graph input to $PATH$, consider a graph with $|V|$ copies of the vertex set V. You'll have to figure out what undirected edges to put in this graph, and what value of k to use. Be sure to include a proof of correctness of your mapping reduction, and a proof that the reduction only uses $O(\log n)$ space.)

4. Let

$$CYCLE = \{< G > | \ G \text{ has a directed cycle}\}.$$

   Prove that $CYCLE$ is NL-complete. (Hint: see the hint for Problem 3.)