CSE 431: Theory of Computation
Assignment #4
April 28, 2000
due: Friday, May 5

In all problems, you should assume that integers are given in binary, vertices in a graph $G = (V, E)$ are represented as binary numbers from 1 to $|V|$, and E is given as a list of pairs of such binary numbers.

1. Problem 8.11.

2. Let

$$USP = \{< G, s, t, k >| \ G \text{ is an undirected graph with a path of length at most } k \text{ between vertices } s \text{ and } t\}.$$

(USP stands for "undirected shortest path".) Show that $USP \in NL$.

3. Let

$$COMPOSITE = \{< k >| \ k \geq 2 \text{ is a composite integer}\}.$$

Show that $COMPOSITE \in NP$. Determine and explain a precise upper bound on the running time of your NTM, e.g., $\Theta(n^2), \Theta(n^4)$, etc., where n is of course the length of the binary encoding of k.

4. Show that the language MAX-CUT of Problem 7.23 is in NP. Determine and explain a precise upper bound on the running time of your NTM.

5. Let M be a Turing machine, either deterministic or nondeterministic. Suppose that M has space complexity $s(n) \geq \log_2 n$. What is the maximum running time of M as a function of $s(n)$? Justify your answer. (Hint: Recall that M must halt on all computation branches and all inputs.)