CSE 431: Theory of Computation
Assignment #2
April 7, 2000
due: Friday, April 14

1. Exercise 4.5. The language should be

$$\{< A >|\ A \text{ is a DFA and } L(A) \text{ is an infinite language}\}.$$

2. Exercise 4.7.

3. Exercise 4.8.

4. (This problem will make your head hurt, but it and the next problem will help ensure that you understand the details of the proof of Theorem 4.9.)

   Sipser uses the $< \ldots >$ notation to indicate encodings into the appropriate input alphabet. Part of the description of the Turing machine D says to run H on input $< M, < M >>$. But there are 3 different Turing machines over whose alphabets we might be doing these encodings.

   (a) Over whose input alphabet is the inner $< M >$ being encoded? Justify your answer.

   (b) Over whose input alphabet is the outer $< M, < M >>$ being encoded? Justify your answer.

5. This problem will explain the origin of the name "The Halting Problem", because you will show that the problem of deciding whether a Turing machine halts is unsolvable. Recall that a Turing machine is said to *loop* if and only if it does not halt. Let

$$L = \{< M, w >|\ M \text{ is a Turing machine that loops on input } w\}.$$

   (a) Give a direct proof, in the style of the proof of Theorem 4.9, that L is undecidable. You may not use a Chapter 5 style reduction for this problem.

   (b) Now prove that L is not Turing-recognizable.