Asymptotic Analysis

Additional Examples

Design and Analysis of Algorithms I

Example #1

Claim : $2^{n+10} = O(2^n)$

Proof : need to pick constants c, n_0 such that

$$(*) \quad 2^{n+10} \leq c \cdot 2^n \quad n \geq n_0$$

Note : $2^{n+10} = 2^{10} \times 2^n = (1024) \times 2^n$

So if we choose $c = 1024, n_0 = 1$ then (*) holds.

Q.E.D

Example #2

<u>Claim</u> : $2^{10n} \neq O(2^n)$

<u>Proof</u> : by contradiction. If $2^{10n} = O(2^n)$ then there exist constants $c, n_0 > 0$ such that

$$2^{10n} \leq c \cdot 2^n \quad n \geq n_0$$

But then [cancelling 2^n]

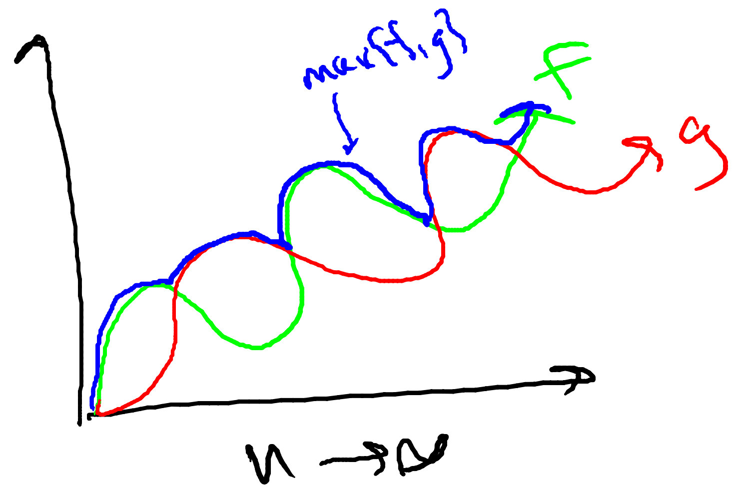
$$2^{9n} \leq c \quad \forall n \geq n_0$$

Which is certainly false. Q.E.D

Example #3

<u>Claim</u> : for every pair of (positive) functions f(n), g(n),

$$max\{f, g\} = \theta(f(n) + g(n))$$



Example #3 (continued)

Proof : $max\{f, g\} = \theta(f(n) + g(n))$

For every n, we have

$$max\{f(n), g(n)\} \leq f(n) + g(n)$$

And

$$2 * max\{f(n), g(n)\} \geq f(n) + g(n)$$

Thus $$\frac{1}{2} * (f(n) + g(n)) \leq max\{f(n), g(n)\} \leq f(n) + g(n) \quad \forall n \geq 1$$

$$=> max\{f, g\} = \theta(f(n) + g(n)) \quad [where \;\; n_0 = 1, c_1 = 1/2, c_2 = 1]$$

Q.E.D

Tim Roughgarden