CSE 417: Algorithms and Computational Complexity **Winter 2012**

Homework 7

Due Wednesday, 3/7/12

Remember: submit each problem, including extra credit, on *its own page.*

Problems

1. (10 pts) Recall from class the reduction from 3-SAT to $INDEPENDENT$-SET. Apply that reduction to the following instance of 3-SAT:

$$(x_1 \vee x_2 \vee \bar{x_3}) \wedge (\bar{x_1} \vee x_4 \vee x_2) \wedge (x_3 \vee \bar{x_1} \vee \bar{x_2})$$

   Namely:

   (a) Show the graph produced by the reduction and an integer k, which should correspond to the size of the independent set you are looking for. Show the correspondence between the literals and the nodes of the graph as detailed in the reduction.

   (b) Show a satisfying assignment for the formula and use it to construct a solution for the $INDEPENDENT$-SET instance.

2. (10 pts) Describe the error in the following fallacious "proof" that $P \neq NP$. Consider an algorithm for SAT: "On input ϕ, try all possible assignments to the variables. Output YES if any satisfy ϕ." This algorithm clearly requires exponential time. Thus SAT has exponential time complexity. Therefore SAT is not in P. Because SAT is in NP, it must be true that P is not equal to NP.

3. (15 pts)We know from class that if we have a polynomial time algorithm for a search problem, we get a polynomial time algorithm for the corresponding decision problem. In this problem, you will prove the converse for the decision problem SAT: Assume that we have access to a polynomial time algorithm that, given a Boolean formula ϕ, decides whether ϕ is satisfiable or not. Using this algorithm as a black box, give a polynomial time algorithm that produces a satisfying assignment for ϕ if it is satisfiable, or outputs "Not Satisfiable" if ϕ is not.

   Hint: Setting a variable x_i to TRUE or FALSE in a formula ϕ on n variables reduces ϕ to a new formula ϕ' on $n-1$ variables. What can we say about the satisfiability of ϕ' in terms of the satisfiability of ϕ?

Extra Credit

1. (15 pts) Chapter 8, Problem 5.