CSE 417: Algorithms and Computational Complexity

4: Dynamic Programming, I
Fibonacci

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Lecture 12

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Some Algorithm Design Techniques, I

- General overall idea
  - Reduce solving a problem to a smaller problem or problems of the same type
- Greedy algorithms
  - Used when one needs to build something a piece at a time
  - Repeatedly make the **greedy** choice - the one that looks the best right away
    - e.g. closest pair in TSP search
  - Usually fast if they work (but often don't)

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Some Algorithm Design Techniques, II

- Divide & Conquer
  - Reduce problem to one or more sub-problems of the same type
  - Typically, each sub-problem is at most a constant fraction of the size of the original problem
    - e.g. Mergesort, Binary Search, Strassen's Algorithm, Quicksort (kind of)

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Some Algorithm Design Techniques, III

- Dynamic Programming
  - Give a solution of a problem using smaller sub-problems, e.g. a recursive solution
  - Useful when the same sub-problems show up again and again in the solution

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"Dynamic Programming"

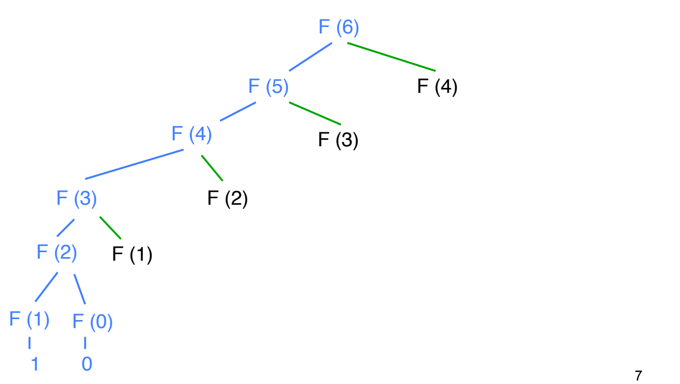
Program — A plan or procedure for dealing with some matter

– Webster's New World Dictionary

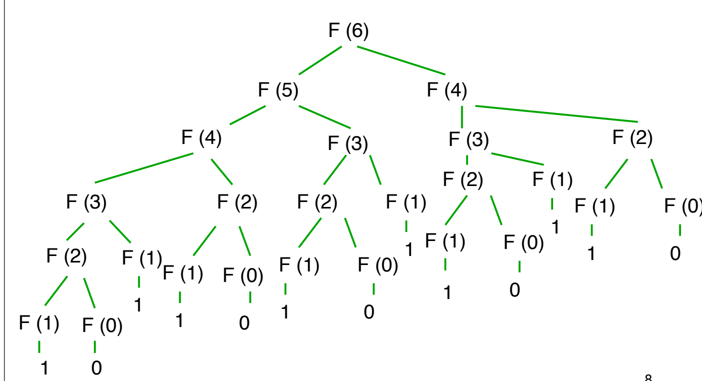
A simple case: Computing Fibonacci Numbers

- Recall $F_n=F_{n-1}+F_{n-2}$ and $F_0=0$, $F_1=1$
- Recursive algorithm:
  - Fibo(n)
    if n=0 **then return**(0)
    else if n=1 **then return**(1)
    else return(Fibo(n-1)+Fibo(n-2))

Call tree - start



Full call tree



Memo-ization (Caching)

- Remember all values from previous recursive calls
- Before recursive call, test to see if value has already been computed
- Dynamic Programming
  - Convert memo-ized algorithm from a recursive one to an iterative one
    (top-down → bottom-up)

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Fibonacci - Memo-ized Version

initialize: F[i] ← undefined for all i
F[0] ← 0
F[1] ← 1
FiboMemo(n):
  if(F[n] undefined) {
    F[n] ← FiboMemo(n-2)+FiboMemo(n-1)
  }
  return(F[n])

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Fibonacci - Dynamic Programming Version

FiboDP(n):
  F[0] ← 0
  F[1] ← 1
  for i=2 **to** n **do**
    F[i]] ← F[i-1]+F[i-2]
  endfor
  return(F[n])

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Dynamic Programming

- Useful when
  - same recursive sub-problems occur repeatedly
  - Can anticipate the parameters of these recursive calls
  - The solution to whole problem can be figured out with**out** knowing the internal details of how the sub-problems are solved
    - principle of optimality

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