CSE 417: Algorithms and Computational Complexity

7,8: Dyn. Programming, IV String Edit Distance

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Sequence Comparison: Edit Distance

- Given:
  - Two strings of characters $A=a_1\ a_2\ ...\ a_n$ and $B=b_1\ b_2\ ...\ b_m$
- Find:
  - The minimum number of edit steps needed to transform A into B where an edit can be:
  - insert a single character
  - delete a single character
  - substitute one character by another

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Applications

- "diff" utility – where do two files differ
- Version control & patch distribution – save/send only changes
- Molecular biology
  - Similar sequences often have similar origin and function
  - Similarity often recognizable despite millions or billions of years of evolutionary divergence

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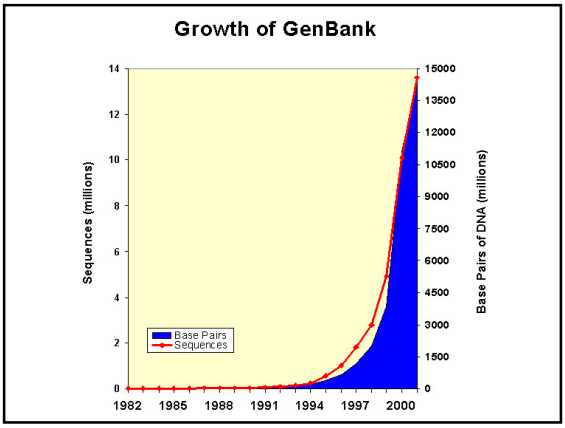

Recursive Solution

- Sub-problems: Edit distance problems for all prefixes of A and B that don't include all of both A and B
- Let D(i,j) be the number of edits required to transform $a_1\ a_2\ ...\ a_i$ into $b_1\ b_2\ ...\ b_j$
- Clearly D(0,0)=0

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Computing D(n,m)

- Imagine how best sequence handles the last characters a_n and b_m
- If best sequence of operations
  - deletes a_n then D(n,m)=D(n-1,m)+1
  - inserts b_m then D(n,m)=D(n,m-1)+1
  - replaces a_n by b_m then D(n,m)=D(n-1,m-1)+1
  - matches a_n and b_m then D(n,m)=D(n-1,m-1)

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Recursive algorithm D(n,m)

```
if n=0 then
    return (m)
elseif m=0 then
    return(n)
else
    if a_n=b_m then
        replace-cost=0
    else
        replace-cost=1
    endif
    return(min{ D(n-1, m) + 1,
                D(n, m-1) +1,
                D(n-1, m-1) + replace-cost})
```

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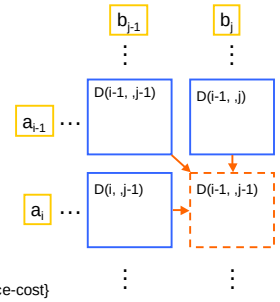
Dynamic Programming

```
for j = 0 to m;  D(0,j) ← j; endfor
for i = 1 to n;  D(i,0) ← i; endfor
for i = 1 to n
   for j = 1 to m
      if a_i=b_j then
         replace-cost ← 0
      else
         replace-cost ← 1
      endif
      D(i,j) ←  min { D(i-1, j) + 1,
                      D(i, j-1) +1,
                      D(i-1, j-1) + replace-cost}
   endfor
endfor
```



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Example run with AGACATTG and GAGTTA

		A 1	G 2	A 3	C 4	A 5	T 6	T 7	G 8
	0								
G 1									
A 2									
G 3									
T 4									
T 5									
A 6									

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Example run with AGACATTG and GAGTTA

		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2								
G	3								
T	4								
T	5								
A	6								

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Example run with AGACATTG and GAGTTA

		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2	1	2	1					
G	3								
T	4								
T	5								
A	6								

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Example run with AGACATTG and GAGTTA

		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2	1	2	1	2	3	4	5	6
G	3	2	1	2	2	3	4	5	5
T	4								
T	5								
A	6								

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Example run with AGACATTG and GAGTTA

		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2	1	2	1	2	3	4	5	6
G	3	2	1	2	2	3	4	5	5
T	4	3	2	2	3	3	3	4	5
T	5	4	3	3	3	4	3	3	4
A	6	5	4	3	4	3	4	4	4

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Example run with AGACATTG and GAGTTA

		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2	1	2	1	2	3	4	5	6
G	3	2	1	2	2	3	4	5	5
T	4	3	2	2	3	3	3	4	5
T	5	4	3	3	3	4	3	3	4
A	6	5	4	3	4	3	4	4	4

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Example run with AGACATTG and GAGTTA

		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2	1	2	1	2	3	4	5	6
G	3	2	1	2	2	3	4	5	5
T	4	3	2	2	3	3	3	4	5
T	5	4	3	3	3	4	3	3	4
A	6	5	4	3	4	3	4	4	4

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Reading off the operations

- Follow the sequence and use each color of arrow to tell you what operation was performed.

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