CSE 417: Algorithms and Computational Complexity

Dealing with NP-completeness

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Paul Beame

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What to do if the problem you want to solve is NP-hard

- You might phrase your problem too generally
  - e.g., in practice, the graphs that actually arise are far from arbitrary
    - maybe they have some special characteristic that allows you to solve the problem in your special case
      - for example the Clique problem is easy on "interval graphs".
    - search the literature to see if special cases already solved

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What to do if the problem you want to solve is NP-hard

- Try to find an approximation algorithm
  - Maybe you can't get the size of the best Vertex Cover but you can find one within a factor of 2 of the best
    - Given graph G=(V,E), start with an empty cover
    - **While** there are still edges in E left
      - **Choose** an edge e={u,v} in E and add both u and v to the cover
      - Remove all edges from E that touch either u or v.
    - Edges chosen don't share any vertices so optimal cover size must be at least # of edges chosen

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What to do if the problem you want to solve is NP-hard

- Try to find an approximation algorithm
  - Recent research has classified problems based on what kinds of approximations are possible if **$P \neq NP$**
    - Best: $(1+\varepsilon)$ factor for any $\varepsilon>0$.
      - packing and some scheduling problems, TSP in plane
    - Some fixed constant factor > 1, e.g. **2, 3/2, 100**
      - Vertex Cover, TSP in space, other scheduling problems
    - $\Theta(\log n)$ factor
      - Set Cover, Graph Partitioning problems
    - Worst: $\Omega(n^{1-\varepsilon})$ factor for any $\varepsilon>0$
      - Clique, Independent-Set, Coloring

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What to do if the problem you want to solve is NP-hard

- Try an algorithm that is provably fast "on average".
  - To even try this one needs a model of what a typical instance is.
  - Typically, people consider "random graphs"
    - e.g. all graphs with a given # of edges are equally likely
  - Problems:
    - real data doesn't look like the random graphs
    - distributions of real data aren't analyzable

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What to do if the problem you want to solve is NP-hard

- Try to search the space of possible hints in a more efficient way and hope it is quick enough
  - e.g. **back-tracking search**
    - For Satisfiability there are 2^n possible truth assignments
    - If we set the truth values one-by-one we might be able to figure out whole parts of the space to avoid,
      - e.g. After setting $x_1 \leftarrow 1$, $x_2 \leftarrow 0$ we don't even need to set x_3 or x_4 to know that it won't satisfy $(\neg x_1 \vee x_2) \wedge (\neg x_2 \vee x_3) \wedge (x_4 \vee \neg x_3) \wedge (x_1 \vee \neg x_4)$
    - For Satisfiability this seems to run in times like $2^{n/20}$ on typical hard instances.
    - Related technique: **branch-and-bound**

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What to do if the problem you want to solve is NP-hard

- Use heuristic algorithms and hope they give good answers
  - No guarantees of quality
  - Many different types of heuristic algorithms

  - Many different options, especially for optimization problems, such as TSP, where we want the best solution.
    - We'll mention several on following slides

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Heuristic algorithms for NP-hard problems

- **local search** for optimization problems
  - need a notion of two solutions being neighbors
  - Start at an arbitrary solution **S**
  - While there is a neighbor **T** of **S** that is better than **S**
    - **S←T**
- Usually fast but often gets stuck in a local optimum and misses the global optimum
  - With some notions of neighbor can take a long time in the worst case

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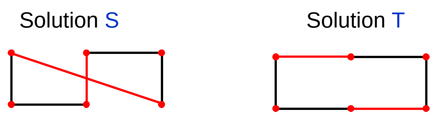
e.g., Neighboring solutions for TSP



Two solutions are neighbors iff there is a pair of edges you can swap to transform one to the other

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Heuristic algorithms for NP-hard problems

- **randomized local search**
  - start local search several times from random starting points and take the best answer found from each point
    - **more expensive than plain local search but usually much better answers**
- **simulated annealing**
  - like local search but at each step sometimes move to a worse neighbor with some probability
    - probability of going to a worse neighbor is set to decrease with time as, presumably, solution is closer to optimal
    - helps avoid getting stuck in a local optimum but often **slow to converge** (much more expensive than randomized local search)
    - analogy with slow cooling to get to lowest energy state in a crystal (or in forging a metal)

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Heuristic algorithms for NP-hard problems

- **genetic algorithms**
  - view each solution as a **string** (analogy with DNA)
  - maintain a **population of good solutions**
  - allow **random mutations** of single characters of individual solutions
  - **combine two solutions** by taking part of one and part of another (analogy with crossover in sexual reproduction)
  - get rid of solutions that have the worst values and make multiple copies of solutions that have the best values (analogy with natural selection -- survival of the fittest).

  - **little evidence that they work well and they are usually very slow**
    - **as much religion as science**

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Heuristic algorithms

- **artificial neural networks**
  - based on very elementary model of human neurons
  - **Set up a circuit of artificial neurons**
    - each artificial neuron is an analog circuit gate whose computation depends on a set of **connection strengths**
  - **Train the circuit**
    - Adjust the connection strengths of the neurons by giving many positive & negative training examples and seeing if it behaves correctly
  - **The network is now ready to use**

  - **useful for ill-defined classification problems such as optical character recognition but not typical cut & dried problems**

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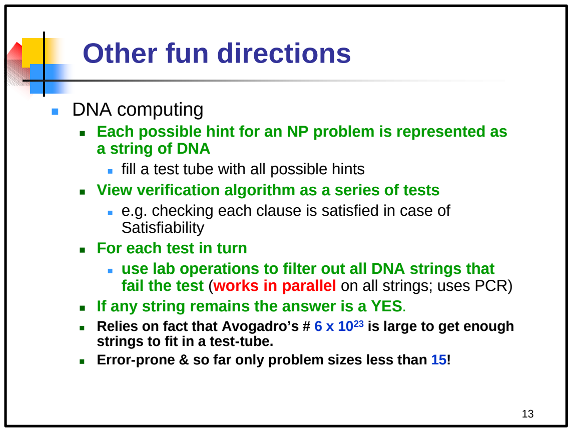
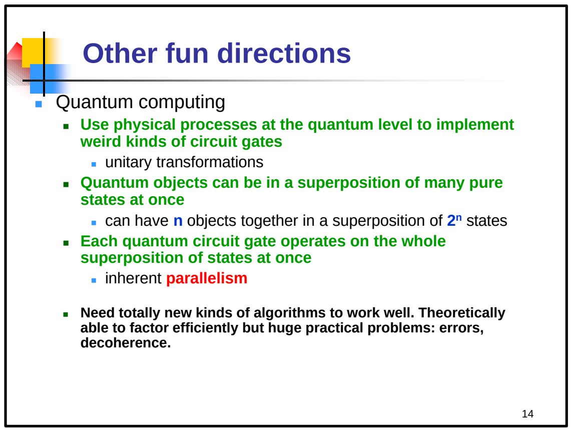


Other fun directions

- DNA computing
  - **Each possible hint for an NP problem is represented as a string of DNA**
    - fill a test tube with all possible hints
  - **View verification algorithm as a series of tests**
    - e.g. checking each clause is satisfied in case of Satisfiability
  - **For each test in turn**
    - **use lab operations to filter out all DNA strings that fail the test** (**works in parallel** on all strings; uses PCR)
  - **If any string remains the answer is a YES**.
  - **Relies on fact that Avogadro's # 6×10^{23} is large to get enough strings to fit in a test-tube.**
  - **Error-prone & so far only problem sizes less than 15!**



Other fun directions

- Quantum computing
  - **Use physical processes at the quantum level to implement weird kinds of circuit gates**
    - unitary transformations
  - **Quantum objects can be in a superposition of many pure states at once**
    - can have **n** objects together in a superposition of 2^n states
  - **Each quantum circuit gate operates on the whole superposition of states at once**
    - inherent **parallelism**
  - **Need totally new kinds of algorithms to work well. Theoretically able to factor efficiently but huge practical problems: errors, decoherence.**