Intel Division Bug

- Scroll back to Autumn 1994 …

The beginning…

Thomas Nicely of Lynchburg College discovered that he got the wrong answers with his Pentium, and posted a query on the Internet asking if others had discovered the same thing?

FROM: Dr. Thomas R. Nicely
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RE: Bug in the Pentium FPU

DATE: 30 October 1994

It appears that there is a bug in the floating point unit (numeric coprocessor) of many, and perhaps all, Pentium processors.

In short, the Pentium FPU is returning erroneous values for certain division operations. For example,

1/824633702441.0

is calculated incorrectly (all digits beyond the eighth significant digit are in error). This can be verified in compiled code, an ordinary spreadsheet such as Quattro Pro or Excel, or even the Windows calculator (use the scientific mode), by computing

(824633702441.0)*(1/824633702441.0),

which should equal 1 exactly (within some extremely small rounding error; in general, coprocessor results should contain 19 significant decimal digits). However, the Pentiums tested return

0.9999999996274709702

for this calculation. A similar erroneous value is obtained for x*(1/x) for most values in the interval

00824633702418 <= x <= 824633702449

Lots of Discussion

- Being on the Internet rather than a private communication with Intel or with some small group of colleagues, the posting attracted considerable attention and made it into the national media
- Many researchers jumped in finding dramatic cases:

  x=4195835, y=3145727 and z=x-(x/y)*y

  should equal 0, but on the Pentium z=256
- Immediately IBM quit shipping PCs with Pentiums
- Intel blamed it's radix-4 SRT algorithm and said the error would occur once in 27,000 years of using a spreadsheet (and had been fixed in new chips)

Their Analysis

Failure category and system component	Hard or Soft	FIT rate (per 10^9 device hours)	MTBF (1 in x years)	Rate of significant failure seen by user
16 4-Mbit DRAM parts in a 60Mhz Pentium ™ processor system without ECC	Soft	16,000	7 years	Depends upon where defect occurs and how propagated
Particle defects in Pentium™ processor	Hard	400-500	200-250 years	Depends upon where defect occurs and how propagated
16 4-Mbit DRAM parts in a 60Mhz Pentium ™ processor system with ECC	Soft	160	700 years	Depends upon where defect occurs and how propagated
PC user on spreadsheet running 1,000 independent divides a day on the Pentium™ processor [a]	Hard	3.3	27,000 years	Less frequent than 1 in 27,000 years. Depends upon the way inaccurate result gets used

a. A detailed analysis on divide usage in spreadsheets is provided in Section 6.2.1 .

Are spreadsheet numbers independent?

- A fire storm began as people pointed out the error in this reasoning
  - Spreadsheet use is likely to focus on a small set of numbers
  - When calculations are complete is the answer any good?
  - IBM's analysis showed an error rate of 1 in 15 days
- The people who complained the loudest were scientific computing types, who, naturally, knew what they were talking about
- Eventually, Andy Grove, CEO of Intel issued a statement over Thanksgiving weekend saying that no chip is perfect (true enough) and adding ...

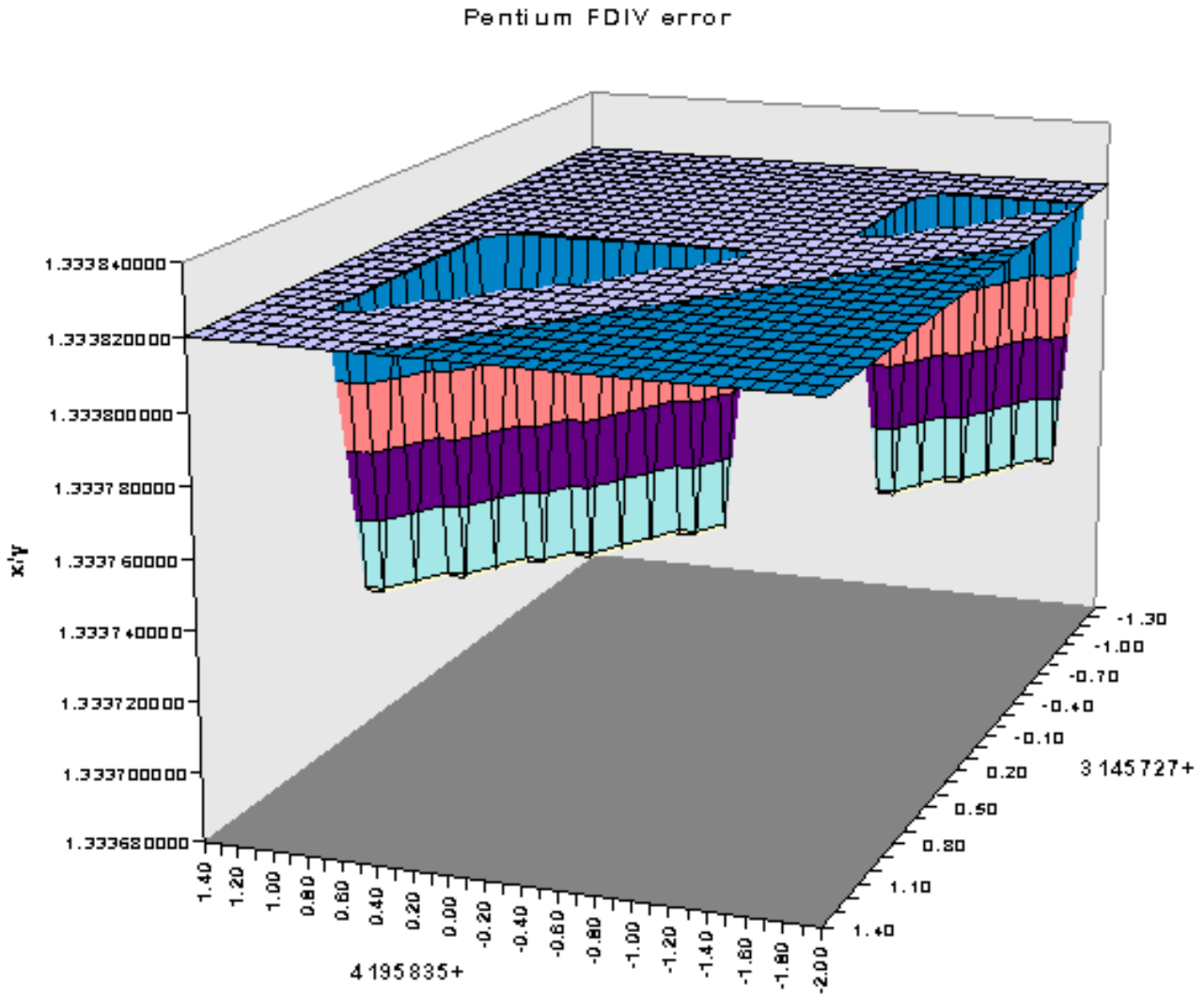
Grove Statement

We would like to find all users of the Pentium processor who are engaged in work involving heavy duty scientific/floating point calculations and resolve their problem in the most appropriate fashion including, if necessary, by replacing their chips with new ones. We don't know how to set precise rules on this so we decided to do it thru individual discussions between each of you and a technically trained Intel person. We set up 800# lines for that purpose. It is going to take us time to work thru the calls we are getting, but we will work thru them. I would like to ask for your patience here.

==

If we think your use warrants a new chip, we'll give you one.

Pictorially



The Radix-4 SRT Divide Algorithm

- So, what went wrong?
- SRT independently invented by Sweeny & Robertson and Tocher in 1958
- Features:
  - Radix 4 means it works on 2 binary digits at a time: faster
  - Non-restoring means quotient bits can be -2, -1, 0, 1, 2
  - Consider division...

$$
\begin{array}{r}
1 \\
\hline
63 \;)\; 818762 \\
\underline{63} \\
188
\end{array}
$$

  - Table driven
  - Bad divisors: 1.0001, 1.0100, 1.0111, 1.1010, 1.1101

Without Details: 7/3

Given:

r = 2 (radix)
D = R(0) = 1.110000 * 2^2
d = 1.100000 * 2^1

Qpos= 00.000000
Qneg= 00.000000

P(0)= 0001.110000 --> q(1)= +1

P(1)= rP(0)-q(1)d
0001.110000
- 0001.100000
0000.010000 --> q(2)= +1

Qpos= 00.000001
Qneg= 00.000000

P(2)= rP(1)-q(2)d
0001.000000
- 0001.100000
1111.100000 --> q(3)= -1

Qpos= 00.000101
Qneg= 00.000000

P(3)= rP(2)-q(3)d
1110.000000
- -0001.100000
1111.100000 --> q(4)= -1

Qpos= 00.010100
Qneg= 00.000001

P(4)= rP(3)-q(4)d
1110.000000
- -0001.100000
1111.100000 (last remainder)

Qpos= 01.010000
Qneg= 00.000101

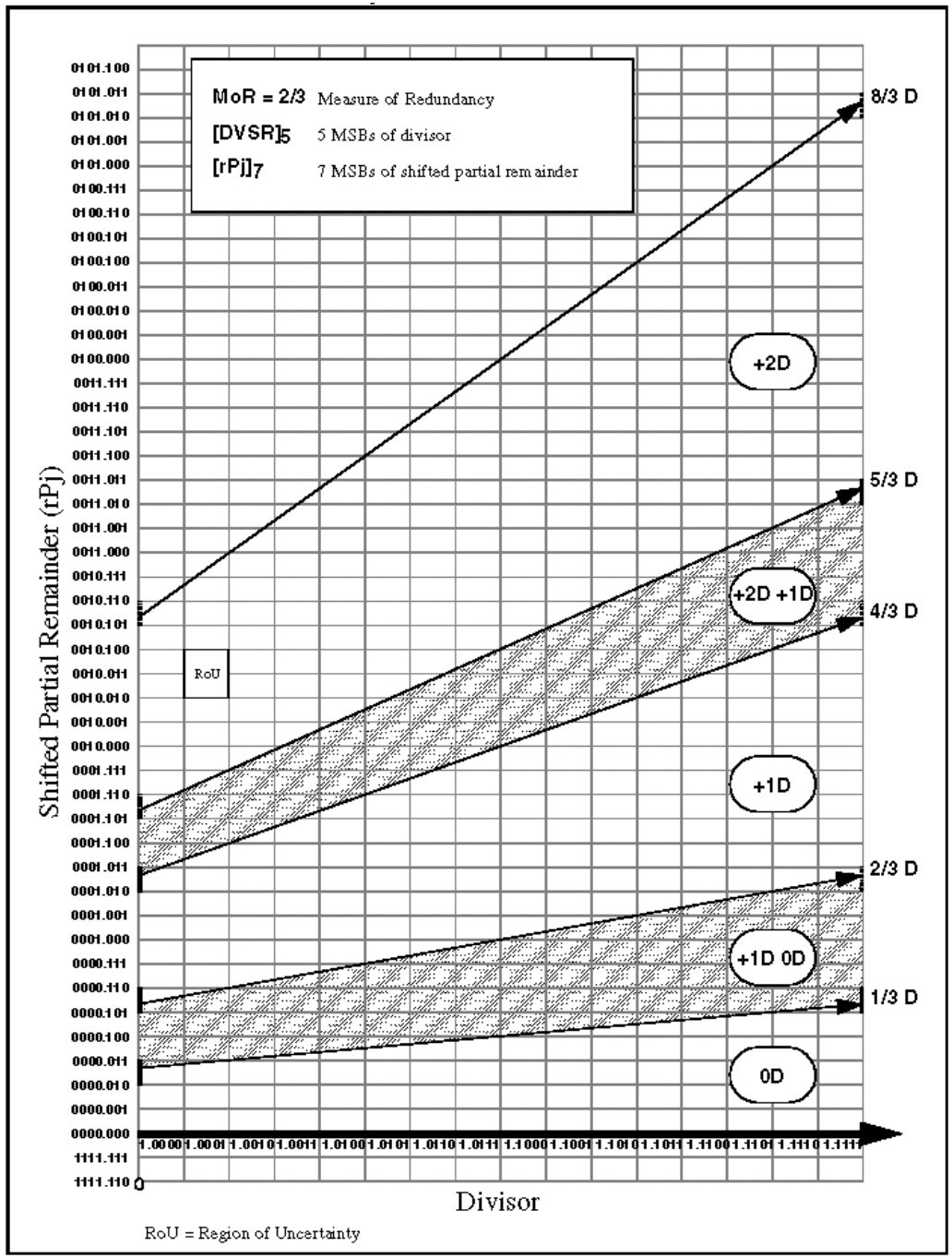
- 1

Q= 01.001010

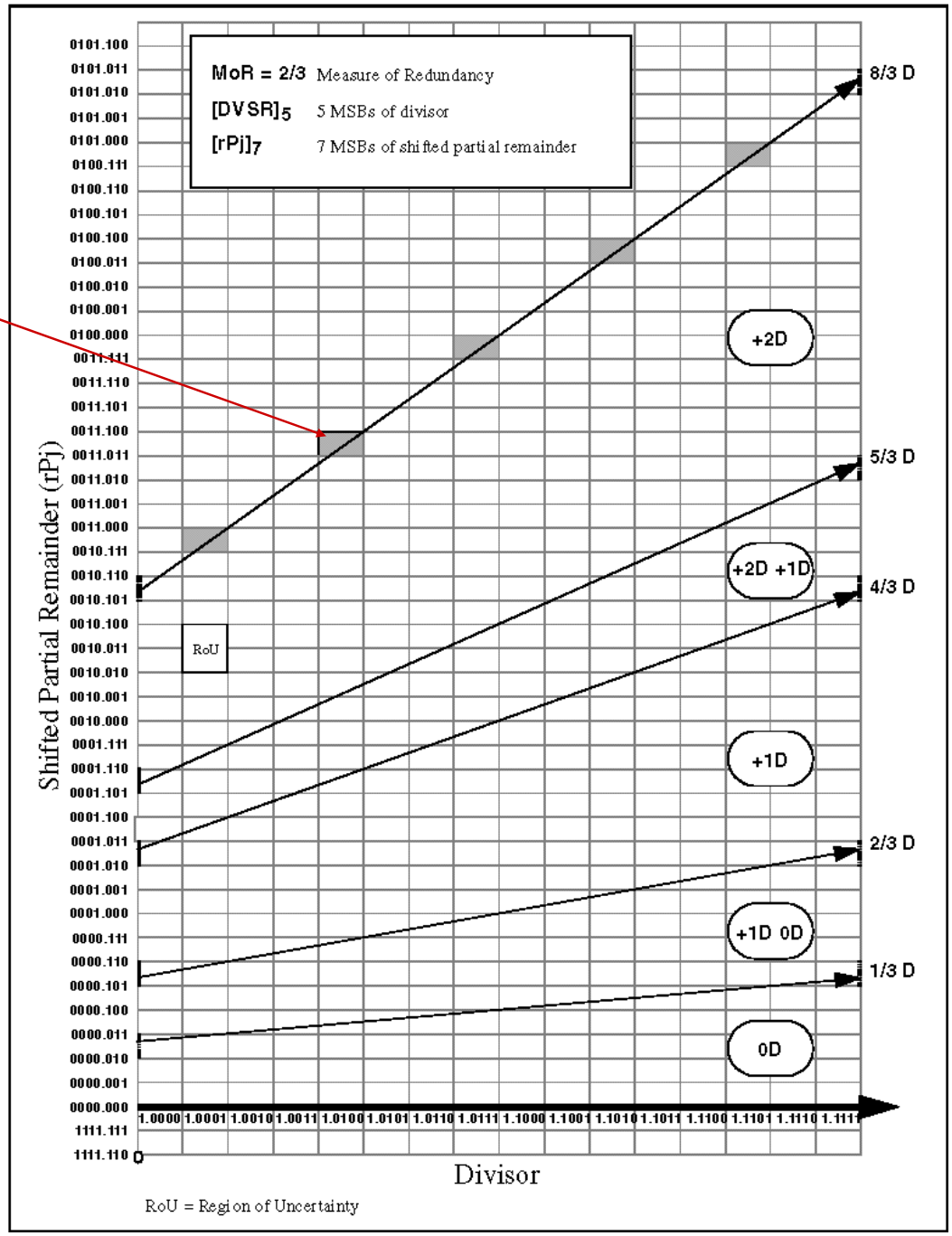
<--P(4) is negative so decrement Q as per algorithm

Q= 01.001010 * 2^1

Half of symmetric table



These entries should contain +2 but they were 0



Resolution

- The matter was settled when, late in December, Intel agreed to replace anyone's chip …

  Intel will exchange the current version of the Pentium processor for an updated version in which this floating-point divide flaw is corrected for any owner who requests it, free of charge anytime during the life of their computer. Just call 1-800-628-8686.

- It is estimated the recall cost Intel $500M, and an untold cost to its reputation

Intel Division Bug … Comic Perspective

Q: How many Pentium designers does it take to screw in a light bulb?

A: 1.99904274017, but that's close enough for non-technical people.

Q: Why didn't Intel call the Pentium the 586?

A: Because they added 486 and 100 on the first Pentium and got 585.999983605.

Q: Complete the following word analogy: *Add* is to *Subtract* as *Multiply* is to

(1) Divide, (2) Round, (3) Random, (4) On a Pentium, all of the above

A: Number 4.

Q: What does the element Pentium decay into?

A: Inert silicon with the emission of a press release.