Name: Key

Email address: _____

CSE 373 Spring 2008: Midterm #1

(closed book, closed notes, NO calculators allowed)

Instructions: Read the directions for each question carefully before answering. We may give partial credit based on the work you **write down**, so if time permits, show your work! Use only the data structures and algorithms we have discussed in class or which were mentioned in the book so far.

Note: For questions where you are drawing pictures, please circle your final answer for any credit. There is one extra page at the end of the exam that you may use for extra space on any problem. If you detach this page it must still be turned in with your exam when you leave.

Good Luck!

Total: 77 points. Time: 50 minutes.

Question	Max Points	Score
1	14	
2	8	
3	13	
4	12	
5	6	
6	18	
7	6	
Total	77	

1. (14 pts) **Big-O**

For each of the functions *f(N)* given below, indicate the tightest bound possible (in other words, giving $O(2^N)$ as the answer to every question is not likely to result in many points). You MUST choose your answer from the following (not given in any particular order), each of which could be re-used (could be the answer for more than one of a) – g)):

$O(N^2)$, $O(N^3 \log N)$, $O(N \log N)$, $O(N)$, $O(N^2 \log N)$, $O(N^5)$, $O(2^N)$, $O(N^3)$, $O(\log N)$, $O(1)$, $O(N^4)$, $O(N^N)$

You do not need to explain your answer.

a) $f(N) = N \log(N^2) + N$ — $O(N \log N)$

$2 \cdot N \log N$

b) $f(N) = 20000^2 + N \log N + N$ — $O(N \log N)$

c) $f(N) = N^{3/2} + N^{3/2}$ — $O(N^2)$

$= 2 \cdot N^{\frac{3}{2}} = 2 \cdot N^{1.5}$

d) $f(N) = 100N + N \log N + N/2$ — $O(N \log N)$

e) $f(N) = N \cdot (\log(N^4) - \log N) + N^2$ — $O(N^2)$

f) $f(N) = N^2 \cdot \log_2(N/2) + N^2$ — $O(N^2 \log N)$

$= N^2 \cdot (\log N - \log 2) + N^2$

g) $f(N) = \frac{1}{2}(N^{1/3}) + \log_2 N$ — $O(N)$

e) $= N \cdot \log \frac{N^4}{N} + N^2$

$= N \log N^3 + N^2$

$= 3 \cdot N \log N + N^2$

2. **(8 pts) Big-Oh and Run Time Analysis:** Describe the worst case running time of the following pseudocode functions in Big-Oh notation in terms of the variable n. ***Showing your work is not required*** (although showing work ***may*** allow some partial credit in the case your answer is wrong – don't spend a lot of time showing your work.). You MUST choose your answer from the following (not given in any particular order), each of which could be re-used (could be the answer for more than one of I. – IV.):

   $O(n^2)$, $O(n^3 \log n)$, $O(n \log n)$, $O(n)$, $O(n^2 \log n)$, $O(n^5)$, $O(2^n)$, $O(n^3)$, $O(\log n)$, $O(1)$, $O(n^4)$, $O(n^n)$

I.
```
void silly(int n) {
   for (int i = 0; i < n; ++i) {
      for (int j = 0; j < i; ++j) {
         System.out.println("j = " + j);
      }
      for (int k = 0; k < n * 3; ++k) {
         System.out.println("k = " + k);
      }
   }
}
```
Runtime: $O(n^2)$

II.
```
void silly(int n, int x, int y) {
   for (int i = 0; i < n; ++i) {
      if (x < y)
         for (int k = 0; k < n * n; ++k){
            System.out.println("k = " + k);
         }
      else
         System.out.println("i = " + i);
   }
}
```
$O(n^3)$

III.
```
void silly(int n) {
   if (n <= 0) return;
   System.out.println("n = " + n);
   silly(n-1);
}
```
$O(n)$

IV.
```
void silly(int n) {
   if (n <= 0) return;
   System.out.println("n = " + n);
   silly(n/2);
}
```
$O(\log n)$

3. (13 pts) **Read each question carefully.** No explanations are necessary, but explanations may yield partial credit. You must solve any summations or recurrences for full credit.

a) (3 pts) What is the minimum and maximum number of nodes *at depth d* in a complete binary tree? (Hint: the root is at depth 0)

Minimum = 1

Maximum = 2^d

b) (3 pts) What is the minimum and maximum number of nodes in the *right subtree* of the root of a perfect binary tree, where the height of the entire tree (not the subtree) is h and h ≥1?

Minimum = $2^h - 1$

Maximum = $2^h - 1$

c) (2 pts) Given an AVL tree containing 31 nodes, after inserting the 32nd value, it is possible that we might have to do three or more rotations in order to maintain the balance property.
(circle one) TRUE or FALSE

d) (2 pts) A complete binary tree satisfies the AVL balance condition at each node.
(circle one) TRUE or FALSE

e) (3 pts) Given an unsorted array of size N, each insert operation will write a value into the array at the next available location. The index of this location is stored in the integer variable `next_loc`. `next_loc` is updated (incremented) after each insert. Once the array is full, the N+1 st insertion will cause a new array of size 2N to be created, and all N+1 values to be copied into the new array. What is the amortized cost of an individual insert operation? (circle one)

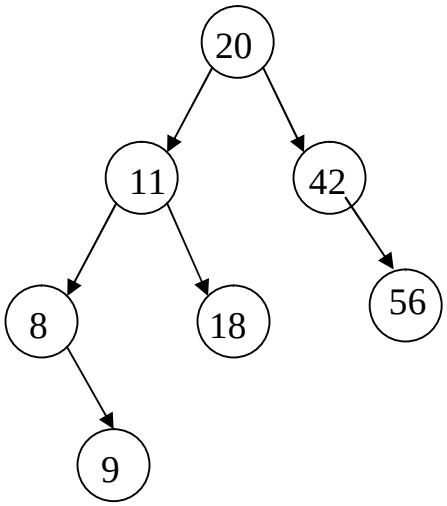
O(N) or $O(N^2)$ or O(1) or O(log N) or O(N log N)

4. (12 pts) **Trees**
   a.) (6 pts) Give traversals of the tree shown below:
   Pre-Order: 20, 11, 8, 9, 18, 42, 56

   Post-Order: 9, 8, 18, 11, 56, 42, 20

   In-Order: 8, 9, 11, 18, 20, 42, 56

   b.) (6 pts) Given the following tree:

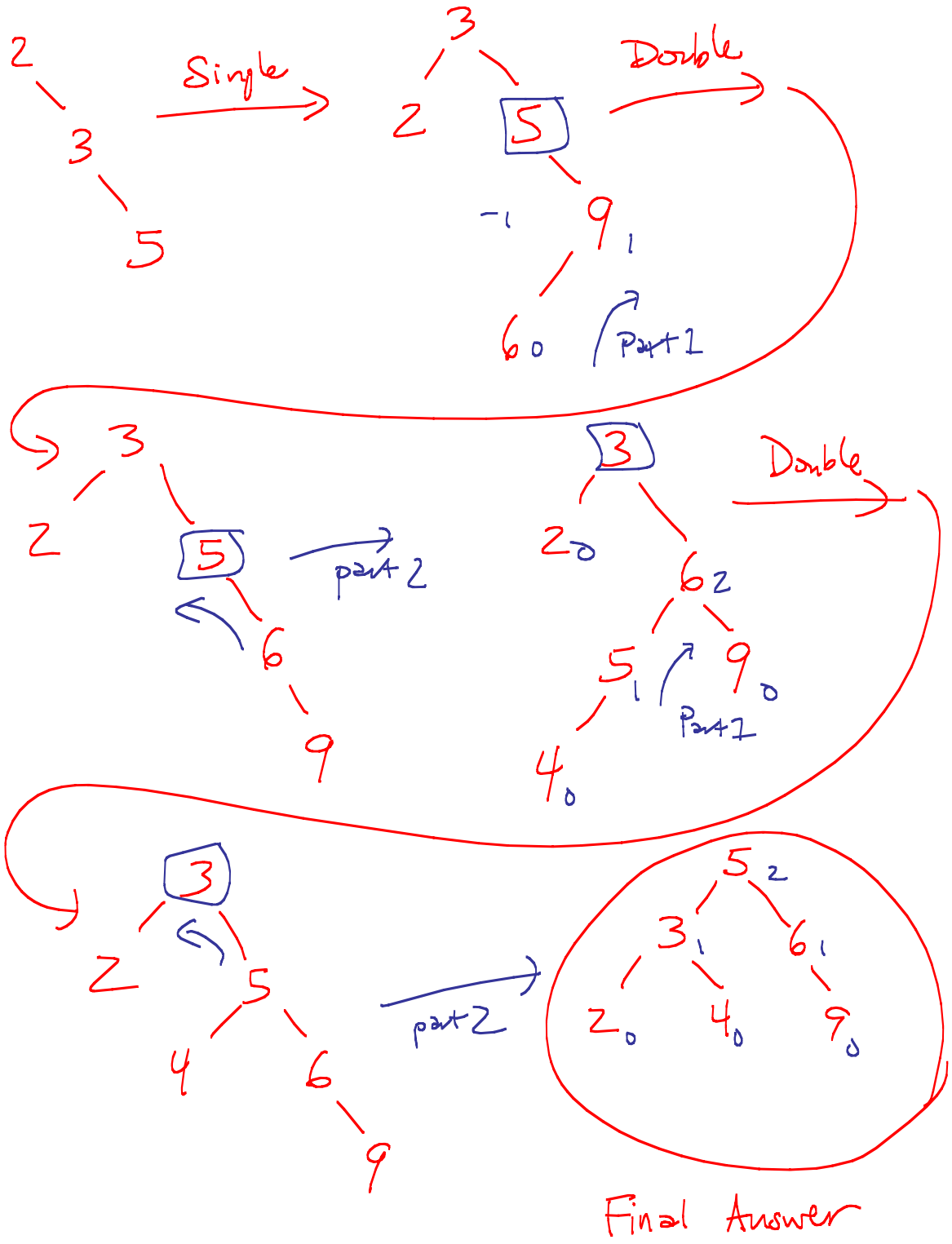


Circle **yes** or **no** to indicate whether the tree above might represent each of the following data structures. If you circle **no, you must describe one way to modify the tree in order to make the answer into a yes.**).

- AVL tree yes no
- Binary Search Tree yes no
- Full Binary Tree yes no

Node 8 + node 42 both need to have left children (all nodes must have zero or 2 children).

5. (6 pts) **AVL Trees** Draw the AVL tree that results from inserting the keys: ~~2, 3, 5, 9, 6,~~ 4 in that order into an initially empty AVL tree. You are only required to show your final tree. However if you draw intermediate trees, ***please circle your final tree for ANY credit.***



6. (18 pts) **Running Time Analysis:** Give the tightest possible upper bound for the *worst case* running time for each of the following in terms of *N*. You MUST choose your answer from the following (not given in any particular order), each of which could be re-used (could be the answer for more than one of a) – f)):

$O(N^2)$, $O(N^3 \log N)$, $O(N \log N)$, $O(N)$, $O(N^2 \log N)$, $O(N^5)$, $O(2^N)$, $O(N^3)$, $O(\log N)$, $O(1)$, $O(N^4)$, $O(N^N)$

****For any credit, you must explain your answer.** Assume that the most time-efficient implementation is used. Assume that all keys are distinct.

a) Delete in a Binary Search Tree of size N

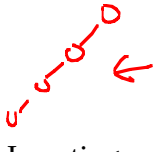
Explanation: Worst case depth of a node = N

Time O(N) to find the node.

a) O(N)

b) Finding the minimum value in a Splay Tree of size N

Explanation: Worst case depth of a node = N



← Time O(N) to find the leftmost node (Splaying up takes time O(N) too)

b) O(N)

c) Inserting a value into an AVL tree of size N, where the value you are inserting is smaller than any other values currently in the tree.

(Doesn't matter)

Explanation: Worst case height of an AVL tree (depth of a node) = O(log N). Time to find proper location = O(log N) + max 1 rotation to fix.

c) O(log N)

d) Pop on a stack containing N elements implemented as a singly-linked list

Explanation: Simply update a constant # of pointers:

temp = top, top = top->next, return temp.

d) O(1)

e) Post-order traversal of a Binary Search Tree containing N elements

Explanation: Must visit each node O(1) times

e) O(N)

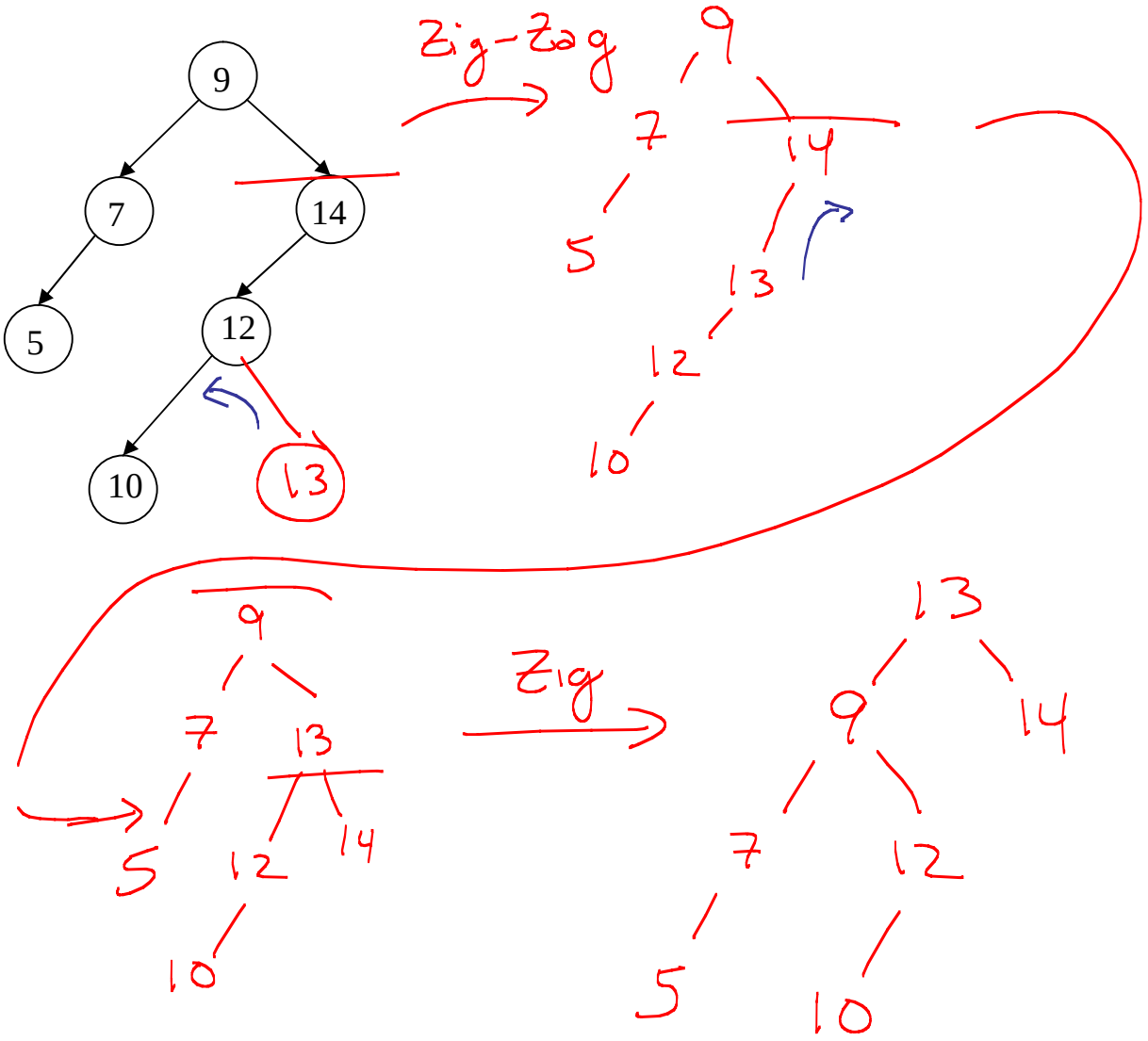
f) In-order traversal of an AVL Tree containing N elements.

Explanation: Same as above. Order of traversal + balance does not matter.

f) O(N)

7. (6 pts) **Splay Trees**

Draw the splay tree that results from inserting the value 13 into the splay tree shown below. Circle your final answer.



Scratch Paper Page (Please Turn In)

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