## Lecture 6

- Logistics
  - TA Office Hour today 3:30 in EEB 003
  - Monday holiday
- Last lecture
  - NAND and NOR and pushing bubbles
  - Logic simplification: Visualization techniques
    - Boolean cubes
- Today's lecture
  - Logic simplification: Visualization techniques
    - Karnaugh maps (K-maps)
    - K-maps with "don't cares"



## The "WHY" slide

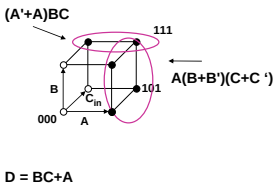
- Karnaugh map (K-map)
  - A visualization tool for logic simplification. It is a flattened version of Boolean cube that allows you to solve even difficult and complex Boolean expression into the minimized form. A convenient tool for you to know for up to 6 input variable expressions (e.g. X = f(A, B, C, D, E, F))
- Don't cares
  - Sometimes the logic output doesn't matter. When we don't care if the output is 0 or 1, rather than assigning random outputs, it is best to denote it as "Don't care." If you learn how to use the "don't care's", you will be able to build even more efficient circuits than without them.



## Recall example using Boolean cube

- On-set is covered by the OR of one 2-D subcube and one 3-D subcube

| A | B | C | D |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |




D = BC+A



## Karnaugh maps (K-map)

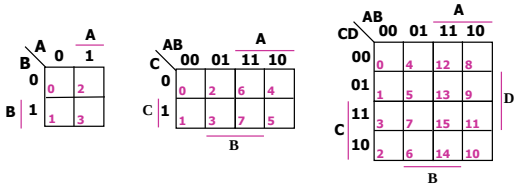
- Flat representation of Boolean cubes
  - Easy to use for 2– 4 dimensions
  - Harder for 5 – 6 dimensions
  - Virtually impossible for >6 dimensions
    - Use CAD tools
- Help visualize adjacencies
  - On-set elements that have one variable changing are adjacent

| | A | B | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 2 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 |




## 2, 3, and 4 dimensional K-maps

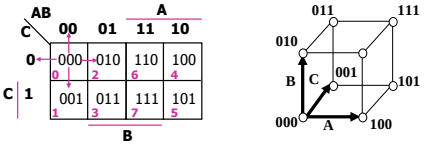
- Uses Gray code: Only one bit changes between cells
  - Example: 00 → 01 → 11 → 10






## Adjacencies

- Wrap–around at edges
  - First column to last column
  - Top row to bottom row



## K-map minimization example: 2 variables

| | A | B | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 2 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 |

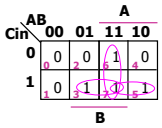

F = B'



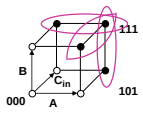
## K-map minimization example: 3 variables

Find the least number of subcubes, each as large as possible, that cover the ON-set
Make sure subcubes contain 1, 2, 4, or 8 items (remember the Boolean cube)

| A | B | Cin | Cout |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |




Cout = AB + BCin + ACin

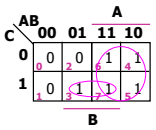




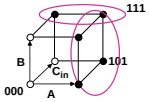

## One more example: 3 variables

Find the least number of subcubes, each as large as possible, that cover the ON-set
Make sure subcubes contain 1, 2, 4, or 8 items (remember the Boolean cube)

| A | B | C | D |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |



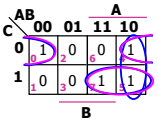

D = A + BC





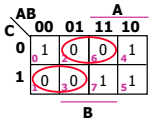

## K-map minimization example: minterms

F(A,B,C) = Σm(0,4,5,7)
= B'C'+AC






## K-map minimization example: complement



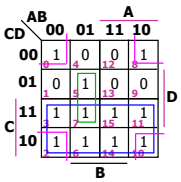

F(A,B,C) = Σm(0,4,5,7)
= B'C'+AC

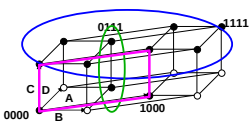
F'(A,B,C) = Σm(1,2,3,6)
= A'C + BC'



## K-map minimization example: 4 variables

- Minimize F(A,B,C,D) = Σm(0,2,3,5,6,7,8,10,11,14,15)
- Answer: F = C+A'BD+B'D'




Find the least number of subcubes, each as large as possible, that cover the ON-set

## K-map minimization examples: on whiteboard

F(A,B,C) = Σm(0,3,6,7)

F(A,B,C) =

F'(A,B,C) =

F(A,B,C,D) = Σm(0,3,7,8,11,15)

F(A,B,C,D) =

F'(A,B,C,D) =

| C \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |



## How about Karnaugh Maps, 6 dimensions?

K-maps become 3D for 5 & 6 variables

AB = 00

| EF \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

AB = 01

| EF \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 1 | 0 | 1 | 1 |
| 10 | 1 | 0 | 0 | 0 |

OUTPUT =
A'BC'D'F' +
CF + BC'D'E

AB = 11

| EF \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 1 | 0 | 1 | 1 |
| 10 | 1 | 0 | 0 | 0 |

AB = 10

| EF \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |



## Incompletely specified functions: Don't cares

- Functions of n inputs have $2^n$ possible configurations
  - Some combinations may be unused
  - Call unused combinations "don't cares"
  - Exploit don't cares during logic minimization
  - Don't care ≠ no output
- Example: A BCD increment-by-1
  - Function F computes the next number in a BCD sequence
    - If the input is $0010_2$, the output is $0011_2$
  - BCD encodes decimal digits 0–9 as $0000_2$–$1001_2$
    - Don't care about binary numbers $1010_2$–$1111_2$



## Truth table for a BCD increment-by-1

| INPUTS | | | | OUTPUTS | | | |
|---|---|---|---|---|---|---|---|
| A | B | C | D | W | X | Y | Z |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | X | X | X | X |
| 1 | 0 | 1 | 1 | X | X | X | X |
| 1 | 1 | 0 | 0 | X | X | X | X |
| 1 | 1 | 0 | 1 | X | X | X | X |
| 1 | 1 | 1 | 0 | X | X | X | X |
| 1 | 1 | 1 | 1 | X | X | X | X |

off-set for W: m0–m6, m9

on-set for W: m7 and m8

Don't care set for W:
We **don't care**
about the output values



## Notation

- Don't cares in canonical forms
  - Three distinct logical sets: {on}, {off}, {don't care}
- Canonical representations of a BCD increment-by-1
  - Minterm expansion
    - W = m7+m8+d10+d11+d12+d13+d14+d15
      = Σm(7,8) + d(10,11,12,13,14,15)
  - Maxterm expansion
    - W = M0•M1•M2•M3•M4•M5•M6•M9•D10•D11•D12•D13•D14•D15
      = ΠM(0,1,2,3,4,5,6,9) • D(10,11,12,13,14,15)
- In K-maps, can treat 'don't cares' as 0s or 1s
  - Depending on which is more advantageous



## Example: with don't cares

- F(A,B,C,D) = Σm(1,3,5,7,9) + d(6,12,13)
  - F = A'D + B'C'D *without using* don't cares
  - F = A'D + C'D *using* don't cares

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | X | 0 |
| 01 | 1 | 1 | X | 1 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 0 | X | 0 | 0 |

A (AB) B (AB) C (CD) D

Assign X == "1"
⇒ allows a 2-cube rather than a 1-cube