

Lecture 10: Quine-McCluskey 2-Level Minimization Algorithm

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Benjamin Ylvisaker

Where We Are

- Last lecture: ROMs, PLAs and PALs, oh my
- This lecture: Quine-McCluskey Minimization
- Next lecture: Multi-Level Logic
- Homework 3 due today; homework 4 out
- In the midst of lab 3

5+ Variable K-Maps are a Pain

- Tedious to draw big K-maps
- Harder to find prime implicants in larger K-maps
- Relatively hard to automate directly

Quine-McCluskey to the Rescue

- Finds the exact same minimum 2-level implementations as K-map minimization
- Table-based method
- Easier to automate

Example Function

- $F(A,B,C,D) = \Sigma m(4,5,6,8,9,10,13) + \Sigma d(0,7,15)$

Step 1

- Translate all minterms and don't care terms to binary
- $F(A,B,C,D) = \Sigma m(4,5,6,8,9,10,13) + \Sigma d(0,7,15)$
- 0100, 0101, 0110, 1000, 1001, 1010, 1101, 0000, 0111, 1111

Step 2

- Sort according to number of 1's in the binary representation
- 0100, 0101, 0110, 1000, 1001, 1010, 1101, 0000, 0111, 1111
- 0000, 0100, 1000, 0101, 0110, 1001, 1010, 1101, 0111, 1111

Step 3

- Put terms in a table, grouped by number of 1's
- 0000, 0100, 1000, 0101, 0110, 1001, 1010, 1101, 0111, 1111
- Column 1
0000

0100
1000

0101
0110
1001
1010

0111
1101

1111

Step 4a

- Compare all pairs of terms from adjacent groups
 - If they match perfectly except for one 0-1 pair, place a term in the next column with a "✓" in that bit position

Column 1	Column 2
0000	0-00
	-000
0100	
1000	010-
	01-0
0101	100-
0110	10-0
1001	
1010	01-1
	-101
0111	011-
1101	1-01
1111	-111
	11-1

Step 4b

- If a term in column 1 can be combined with any other term in column 1 to make a term in column 2, place a check next to it

Column 1	Column 2
0000✓	0-00
	-000
0100✓	
1000✓	010-
	01-0
0101✓	100-
0110✓	10-0
1001✓	
1010✓	01-1
	-101
0111✓	011-
1101✓	1-01
1111✓	-111
	11-1

Step 4a (again)

Column 1	Column 2	Column 3
0000✓	0-00	01--
	-000	
0100✓		-1-1
1000✓	010-	
	01-0	
0101✓	100-	
0110✓	10-0	
1001✓		
1010✓	01-1	
	-101	
0111✓	011-	
1101✓	1-01	
1111✓	-111	
	11-1	

Step 4b (again)

Column 1	Column 2	Column 3
0000✓	0-00	01--
	-000	
0100✓		-1-1
1000✓	010-✓	
	01-0✓	
0101✓	100-	
0110✓	10-0	
1001✓		
1010✓	01-1✓	
	-101✓	
0111✓	011-✓	
1101✓	1-01	
1111✓	-111✓	
	11-1✓	

Step 4c

- Place a * next to and term in column 2 that cannot be combined with another term in column 2 to make a term in column 3

Column 1	Column 2	Column 3
0000√	0-00* -000*	01--
0100√		-1-1
1000√	010-√ 01-0√	
0101√	100-*	
0110√	10-0*	
1001√		
1010√	01-1√ -101√	
0111√	011-√	
1101√	1-01*	
1111√	-111√ 11-1√	

Step 4a (again)

Column 1	Column 2	Column 3	Column 4
0000√	0-00* -000*	01--	
0100√		-1-1	
1000√	010-√ 01-0√		
0101√	100-*		
0110√	10-0*		
1001√			
1010√	01-1√ -101√		
0111√	011-√		
1101√	1-01*		
1111√	-111√ 11-1√		

Step 4b (again)

Column 1	Column 2	Column 3	Column 4
0000√	0-00* -000*	01--	
0100√		-1-1	
1000√	010-√ 01-0√		
0101√	100-*		
0110√	10-0*		
1001√			
1010√	01-1√ -101√		
0111√	011-√		
1101√	1-01*		
1111√	-111√ 11-1√		

Step 4c (again)

Column 1	Column 2	Column 3	Column 4
0000√	0-00* -000*	01--*	
0100√		-1-1*	
1000√	010-√ 01-0√		
0101√	100-*		
0110√	10-0*		
1001√			
1010√	01-1√ -101√		
0111√	011-√		
1101√	1-01*		
1111√	-111√ 11-1√		

Step 5

- List all of the *-ed terms; these are the prime implicants

- 0-00, -000, 100-, 10-0, 1-01, 01--, -1-1

Column 1	Column 2	Column 3	Column 4
0000√	0-00*	01--*	
	-000*		
0100√		-1-1*	
1000√	010-√		
	01-0√		
0101√	100-*		
0110√	10-0*		
1001√			
1010√	01-1√		
	-101√		
0111√	011-√		
1101√	1-01*		
1111√	-111√		
	11-1√		

Step 6

- Build a prime implicant table, with the prime implicants along the left, the minterms (but not the don't cares) along the top, and an x in every cell where the prime implicant covers the minterm

	4	5	6	8	9	10	13
0-00	*						
-000				*			
100-				*	*		
10-0				*		*	
1-01					*		*
01--	*	*	*				
-1-1		*					*

Step 7a

- Find columns (minterms) that only have an x in one row (implicant); "include" these implicants in the function implementation

	4	5	6	8	9	10	13
0-00	*						
-000				*			
100-				*	*		
10-0				*		*	
1-01					*		*
01--	*	*	*				
-1-1		*					*

- {10-0, 01--}

Step 7b

- Remove the included implicants from the table, and any minterms that they cover

	9	13
0-00		
-000		
100-	*	
1-01	*	*
-1-1		*

- {10-0, 01--}

Step 8

- Heuristically include more implicants until all minterms are covered

- | | | |
|------|------|------|
| | 9 | 13 |
| 0-00 | ---- | ---- |
| -000 | ---- | ---- |
| 100- | ---- | ---- |
| 1-01 | ---- | ---- |
| -1-1 | ---- | ---- |

- {10-0, 01-- , 1-01}

Step 9

- Translate included implicants into a minimized sum-of-products form

- {10-0, 01-- , 1-01}

- $F(A,B,C,D) = A-B-D + \neg AB + A-CD$

Now You Try

- $F(A,B,C,D) = \Sigma m(0,3,6,8,9,11,15) + \Sigma d(1,2,4,12)$

- Column 1

```
0000
0001
0010
0100
1000
```

```
0011
0110
1001
1100
```

```
1011
```

```
1111
```

Thank You for Your Attention

- Finish lab 3; start reading lab 4
- Start looking at homework 4
- Continue reading the book