

Lecture 6: Boolean Cubes and Karnaugh Maps

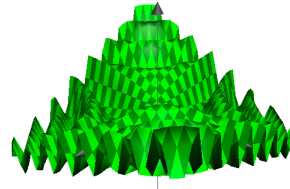
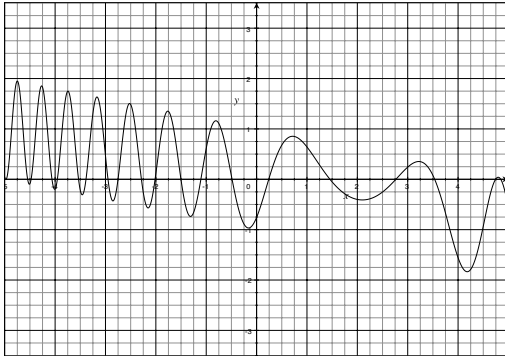
CSE 370, Autumn 2007
Benjamin Ylvisaker

Where We Are

- Last lecture: 2-level implementations and canonical forms
- This lecture: Boolean cubes and K-maps
- Next lecture: K-map minimization
- Homework 1 back today or Wed. 2 due Wed.
- Read lab 2 before the start of your session

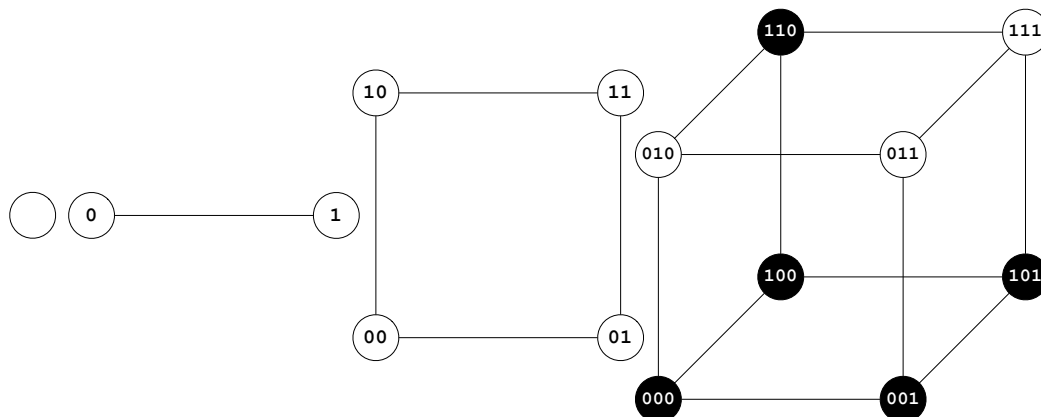
Inspiration

- Visualization of real-valued functions

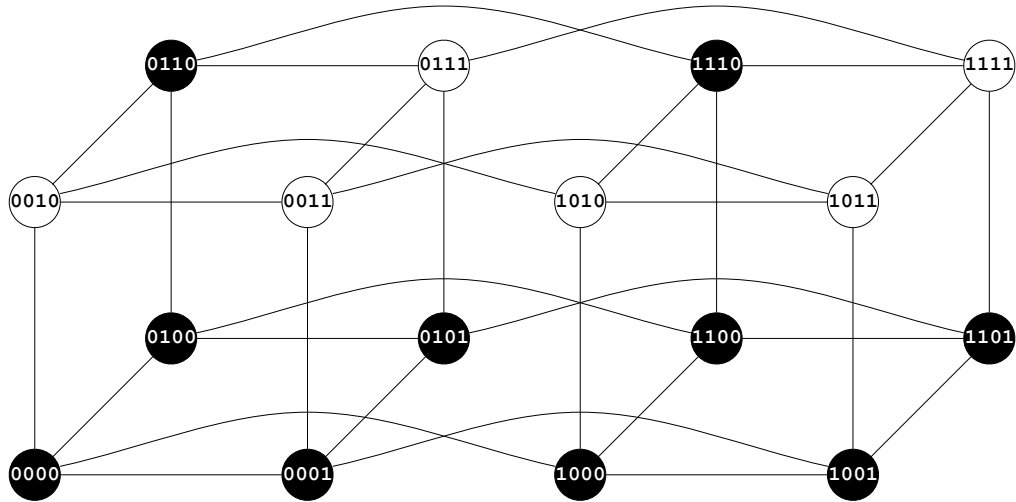


Visualizing Boolean Functions

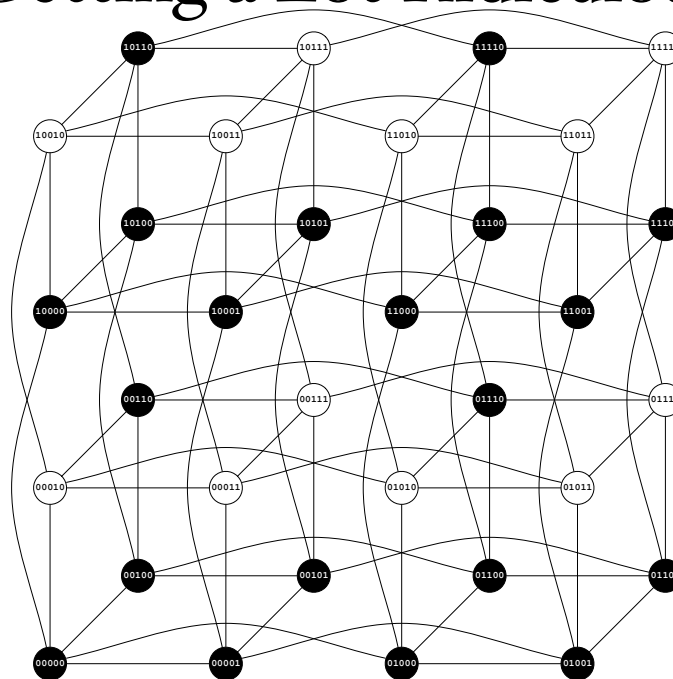
- Generally more input variables
- **Way** fewer possible values per variable



Getting a Little Ridiculous

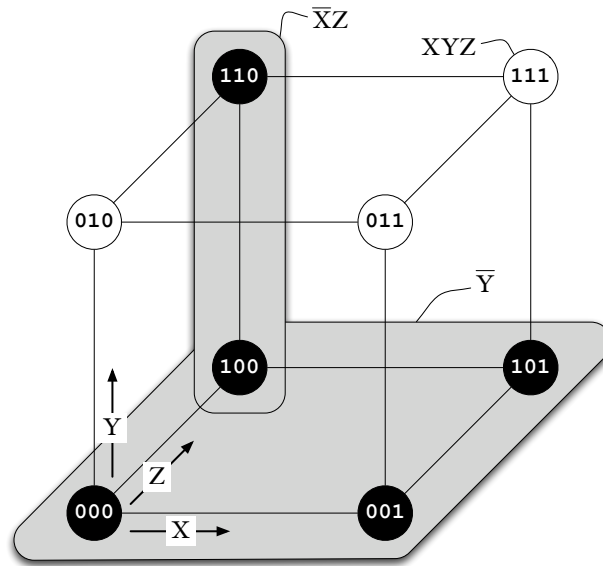


Getting a Lot Ridiculous



The Features of a Cube

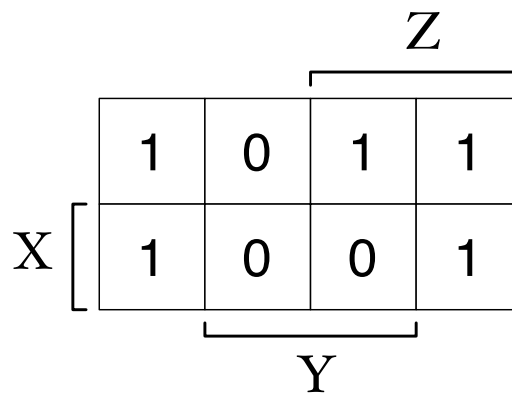
- | Z | Y | X | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |



Karnaugh Maps

- Flattened Boolean cubes

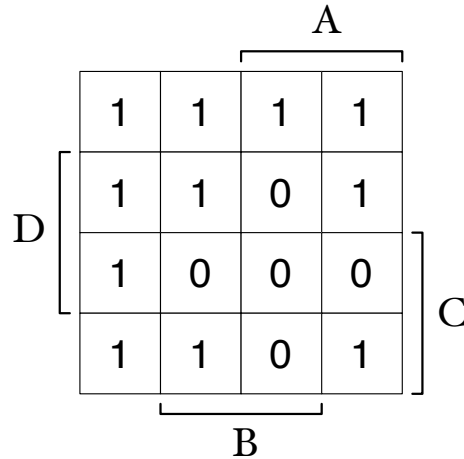
Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



4 Variable Example

- Inverse majority function

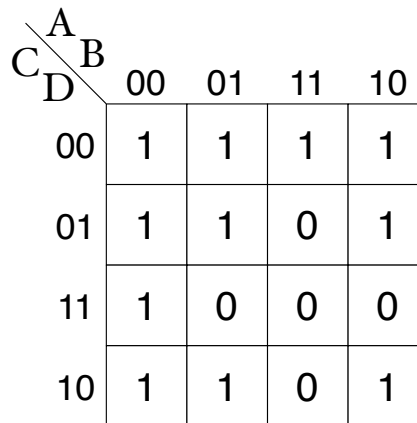
A	B	C	D	F	A	B	C	D	F
0	0	0	0	1	1	0	0	0	1
0	0	0	1	1	1	0	0	1	1
0	0	1	0	1	1	0	1	0	1
0	0	1	1	1	1	0	1	1	0
0	1	0	0	1	1	1	0	0	1
0	1	0	1	1	1	1	0	1	0
0	1	1	0	1	1	1	1	0	0
0	1	1	1	0	1	1	1	1	0



Different Way to Draw a K-Map

- Inverse majority function

A	B	C	D	F	A	B	C	D	F
0	0	0	0	1	1	0	0	0	1
0	0	0	1	1	1	0	0	1	1
0	0	1	0	1	1	0	1	0	1
0	0	1	1	1	1	0	1	1	0
0	1	0	0	1	1	1	0	0	1
0	1	0	1	1	1	1	0	1	0
0	1	1	0	1	1	1	1	0	0
0	1	1	1	0	1	1	1	1	0



Rectangles in K-Maps

- Always a power of 2 on a side
- Can “wrap around” the border
- Can only enclose all 1's or all 0's
 - 1's correspond to product terms
 - 0's correspond to sum terms

		A B			
		00	01	11	10
C D	00	1	1	1	1
	01	1	1	0	1
	11	1	0	0	0
	10	1	1	0	1

Individual Terms

		A B			
		00	01	11	10
C D	00	1	1	1	1
	01	1	1	0	1
	11	1	0	0	0
	10	1	1	0	1

• $\neg C \neg D$

• $\neg A \neg C$

• $\neg A + \neg B + \neg D$

• $\neg B \neg D$

Bad Rectangles

		A	B		
	C	D			
		00	01	11	10
• Non-power of 2 width	00	1	1	1	1
• Non-rectangle	01	1	1	0	1
• Non-power of 2 height	11	1	0	0	0
• Encloses both 0's and 1's	10	1	1	0	1

Numbering the Squares

- Inverse majority function

A	B	C	D	#	A	B	C	D	#
0	0	0	0	0	1	0	0	0	8
0	0	0	1	1	1	0	0	1	9
0	0	1	0	2	1	0	1	0	10
0	0	1	1	3	1	0	1	1	11
0	1	0	0	4	1	1	0	0	12
0	1	0	1	5	1	1	0	1	13
0	1	1	0	6	1	1	1	0	14
0	1	1	1	7	1	1	1	1	15

		A			
		0	4	12	8
		1	5	13	9
D		3	7	15	10
		2	6	14	11
		B			
				C	

Now You Try

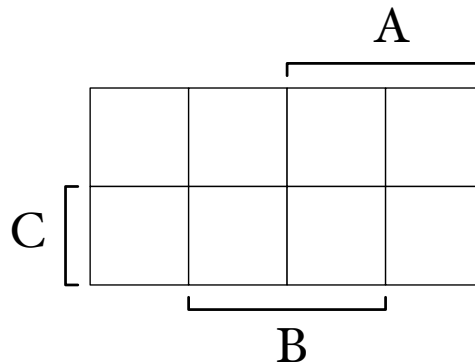
- Multiplexer

A	B	C	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Now You Try

- Multiplexer

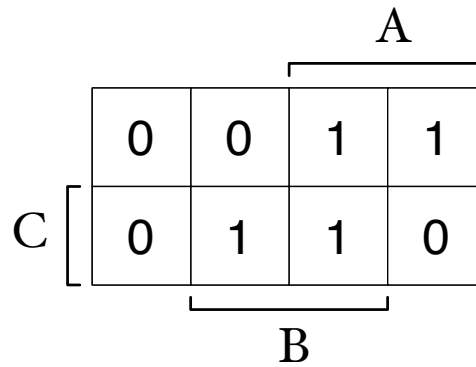
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Now You Try

- Multiplexer

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Thank You for Your Attention

- Pick up your quiz
- Read lab 2
- Continue homework 2
- Continue reading the book