Lecture 5:
2-Level Logic and Canonical Forms

CSE 370, Autumn 2007
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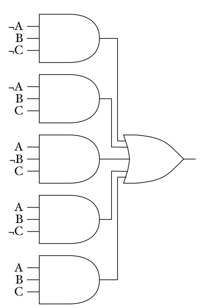
Where We Are

- Last lecture: Truth tables and more functions
- This lecture: 2-level implementations and canonical forms
- Next lecture: Boolean cubes
- Homework 1 in the grading pipeline; start 2
- How was lab 2?
  - Start looking at lab 3
- Tutoring available

Every Function Can Be Implemented in 2 Levels

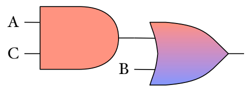
-

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



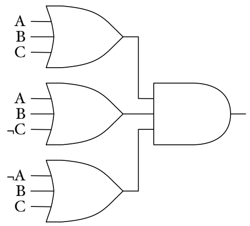
But We Can Be More Clever

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



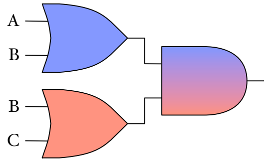
We Can Also Look At the 0's

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



Again With the Cleverness

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



There Are Lots of Ways to Implement a Function

- ... even if we only consider circuits of the 2 level style
- Sometimes we only want one possible representation for a given function
  - Makes it easy to decide when two people (or programs) have the same function
- Canonical forms to the rescue!

Minterms and Maxterms

Row#	A	B	AB	A+B	$\overline{AB}$	$\overline{A+B}$	A⊕B	$\overline{A \oplus B}$
0	0	0	0	0	1	1	0	1
1	0	1	0	1	1	0	1	0
2	1	0	0	1	1	0	1	0
3	1	1	1	1	0	0	0	1

- AB $= \Sigma m(3)$ $= \Pi M(0,1,2)$
  A+B $= \Sigma m(1,2,3)$ $= \Pi M(0)$
  ¬(AB) $= \Sigma m(0,1,2)$ $= \Pi M(3)$
  A⊕B $= \Sigma m(1,2)$ $= \Pi M(0,3)$

Gray Code

- In the coming examples we will use a system called gray code
- Successive numbers differ in exactly 1 bit position
- ```
  0 = 000
  1 = 001
  2 = 011
  3 = 010
  4 = 110
  5 = 111
  6 = 101
  7 = 100
  ```
  ```

Gray Code Successor Function

- Input: Output:

Input:	Output:
000	001
001	011
011	010
010	110
110	111
111	101
101	100
100	000

- We can treat each bit (each column) of the output as its own 3-variable Boolean function
  - The three functions taken together give us the complete successor

Gray Code Successor Function Truth Table

Input:	Output:
000	001
001	011
011	010
010	110
110	111
111	101
101	100
100	000

A	B	C	D	E	F
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	1

Gray Code Successor Function(s) in Minterm Notation

A	B	C	D	E	F
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	1

$D(A,B,C) = \Sigma m(2,5,6,7)$
$E(A,B,C) = \Sigma m(1,2,3,6)$
$F(A,B,C) = \Sigma m(0,1,6,7)$

Order of the variables matters!

$D(A,C,B) = \Sigma m(1,5,6,7)$
$E(A,C,B) = \Sigma m(1,2,3,5)$
$F(A,C,B) = \Sigma m(0,2,5,7)$

Now in Maxterm Notation

A	B	C	D	E	F
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	1

D(A,B,C) = ΠM(0,1,3,4)
E(A,B,C) = ΠM(0,4,5,7)
F(A,B,C) = ΠM(2,3,4,5)

Order of the variables matters!

D(B,A,C) = ΠM(0,1,2,5)
E(B,A,C) = ΠM(0,2,3,7)
F(B,A,C) = ΠM(2,3,4,5)

Binary-Coded Decimal (BCD)

- BCD is an encoding for more directly representing decimal numbers with binary digits
- Each 4 bits represents 1 decimal digit
- Useful in some numerical programs
- 0 = 0000   5 = 0101
  1 = 0001   6 = 0110
  2 = 0010   7 = 0111
  3 = 0011   8 = 1000
  4 = 0100   9 = 1001

BCD to Gray Code Converter

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0

A	B	C	D	E	F	G	H
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

We Can Compact the Table

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0

A	B	C	D	E	F	G	H
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

We Can Compact the Table

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0

A	B	C	D	E	F	G	H
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

$E(A,B,C,D) = \Sigma m(8,9) + \Sigma d(10\text{-}15)$

$E(A,B,C,D) = \Pi M(0\text{-}7)\Pi D(10\text{-}15)$

We Can Compact the Table

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0

A	B	C	D	E	F	G	H
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

$F(A,B,C,D) = \Sigma m(4\text{-}9) + \Sigma d(10\text{-}15)$

$F(A,B,C,D) = \Pi M(0\text{-}3)\Pi D(10\text{-}15)$

We Can Compact the Table

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0

A	B	C	D	E	F	G	H
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

$G(A,B,C,D) = \Sigma m(2\text{-}5) + \Sigma d(10\text{-}15)$

$G(A,B,C,D) = \Pi M(0,1,6\text{-}9) \Pi D(10\text{-}15)$

We Can Compact the Table

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0

A	B	C	D	E	F	G	H
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

$H(A,B,C,D) = \Sigma m(1,2,5,6,9) + \Sigma d(10\text{-}15)$

$H(A,B,C,D) = \Pi M(0,3,4,7,8) \Pi D(10\text{-}15)$

Lots of Representations

- Boolean algebra expressions/functions
- Digital circuit diagrams
- Truth tables
- Minterm and maxterm notation
- Next time: Boolean cubes & Karnaugh maps
- BDDs: {Boolean/Binary} Decision Diagrams
  - Not discussed in 370

Thank You for Your Attention

- Collect your quizzes
- Continue work on homework 2
- Start looking at lab 2
- Continue reading the book