Lecture 3:
All Hail George Boole

CSE 370, Autumn 2007
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Where We Are

- Last lecture: Binary numbers & arithmetic
- This lecture: Boolean algebra
- Next lecture: Playing around w/ Boolean functions
- Homework 1 due Wednesday at the beginning of class
- Lab 1 this week. Read it before the session starts!

Boolean Logic/Algebra

- Notation for writing down precise logical statements (in propositional logic)
- Primitives: true, false, variables
- Connectives: NOT, AND, OR, IMPLIES, ...
- (Almost) all memoryless digital circuits can be seen as Boolean algebra expressions

Why Do We Care?

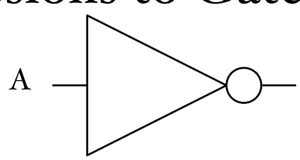
- Understanding Boolean logic helps us design "simpler" circuits, both by hand and automatically
- ((A AND B) OR (NOT A AND B)) AND A
- Equivalent to: A AND B

Lots of Alternative Notations

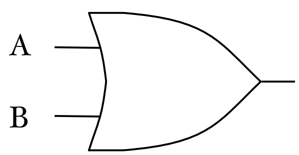
- I will mostly use:
  - ¬A for NOT A
  - A+B for A OR B
  - A•B for A AND B
- Book lists all of the common notations

From Expressions to Gates

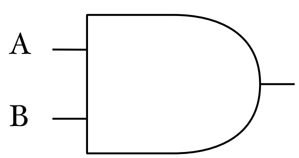
- NOT A



- A OR B



- A AND B



The Useful Theorems

- Several slides of statements of basic facts about Boolean algebra
- Every theorem comes with a "dual"

0 and 1

- X+0=X   X•1=X
- X+1=1   X•0=0

Idempotence

- $X+X=X$ $X \bullet X=X$

Involution

- $\neg\neg X=X$

Complementarity

- $X+\neg X=1$ $\quad X \bullet \neg X=0$

Commutativity

- $X+Y=Y+X$ $\quad X \bullet Y=Y \bullet X$

Associativity

- $(X+Y)+Z = X+(Y+Z) = X+Y+Z$ $\quad$ $(X \bullet Y) \bullet Z = X \bullet (Y \bullet Z) = X \bullet Y \bullet Z$

Distributivity

- $X \bullet (Y+Z) = (X \bullet Y)+(X \bullet Z)$ $\quad$ $X+(Y \bullet Z) = (X+Y) \bullet (X+Z)$

Some Simplifications

- $(X \bullet Y)+(X \bullet \neg Y)=X$ $(X+Y) \bullet (X+\neg Y)=X$
- $X+(X \bullet Y)=X$ $X \bullet (X+Y)=X$
- $(X+\neg Y) \bullet Y=X \bullet Y$ $(X \bullet \neg Y)+Y=X+Y$

Prove Simplification 1

- $(X \bullet Y)+(X \bullet \neg Y) \overset{?}{=} X$ $(X+Y) \bullet (X+\neg Y) \overset{?}{=} X$
  - By distributivity
- $X \bullet (Y+\neg Y) \overset{?}{=} X$ $X+(Y \bullet \neg Y) \overset{?}{=} X$
  - By complementarity
- $X \bullet 1 \overset{?}{=} X$ $X+0 \overset{?}{=} X$
  - By identity
- $X=X$ $X=X$

Prove Simplification 2

- $X+(X \bullet Y) \stackrel{?}{=} X$ and $X \bullet (X+Y) \stackrel{?}{=} X$
  - By identity
- $(X \bullet 1)+(X \bullet Y) \stackrel{?}{=} X$ and $(X+0) \bullet (X+Y) \stackrel{?}{=} X$
  - By distributivity
- $X \bullet (1+Y) \stackrel{?}{=} X$ and $X+(0 \bullet Y) \stackrel{?}{=} X$
  - By identity
- $X \bullet 1 \stackrel{?}{=} X$ and $X+0 \stackrel{?}{=} X$
  - By identity
- $X=X$ and $X=X$

Prove Simplification 3

- $(X+\neg Y) \bullet Y \stackrel{?}{=} X \bullet Y$ and $(X \bullet \neg Y)+Y \stackrel{?}{=} X+Y$
  - By simplification 2
- $(X+\neg Y) \bullet ((Y+\neg Y) \bullet Y) \stackrel{?}{=} X \bullet Y$ and $(X \bullet \neg Y)+((Y \bullet \neg Y)+Y) \stackrel{?}{=} X+Y$
  - By associativity
- $(X+\neg Y) \bullet (Y+\neg Y) \bullet Y \stackrel{?}{=} X \bullet Y$ and $(X \bullet \neg Y)+(Y \bullet \neg Y)+Y \stackrel{?}{=} X+Y$
  - By distributivity
- $((X \bullet Y)+\neg Y) \bullet Y \stackrel{?}{=} X \bullet Y$ and $((X+Y) \bullet \neg Y)+Y \stackrel{?}{=} X+Y$
  - By distributivity
- $(X \bullet Y \bullet Y)+(\neg Y \bullet Y) \stackrel{?}{=} X \bullet Y$ and $(X+Y+Y) \bullet (\neg Y+Y) \stackrel{?}{=} X+Y$
  - By associativity, idempotence and complementarity
- $(X \bullet Y)+0 \stackrel{?}{=} X \bullet Y$ and $(X+Y) \bullet 1 \stackrel{?}{=} X+Y$
  - By operations with 1 and 0
- $X \bullet Y=X \bullet Y$ and $X+Y=X+Y$

DeMorgan's law (or theorem)

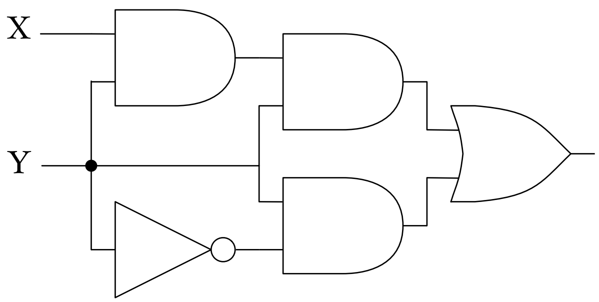
- $\neg(X+Y)=\neg X \bullet \neg Y$ $\neg(X \bullet Y)=\neg X+\neg Y$

Duality

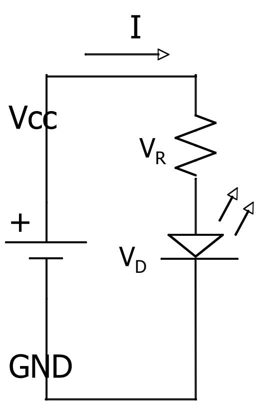
- A Boolean function is just an expression with a name and a "parameter list" of variables used in the expression
  - $f(A,B,C) = (A \bullet B)+C$
- The dual of a function (written $f(A,B,C)^D$) is the function with •'s and +'s swapped and 1's and 0's swapped
  - $f(A,B,C)^D = (A+B) \bullet C$

A Bigger Circuit Diagram

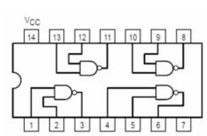
- (X•Y•Y)+(¬Y•Y)



Real Circuits Can Hurt You



- Current flows from higher voltages to lower voltages
- 1=V_{CC}, 0=Gnd
- Must always hook logic chips up to power and ground
- Never connect the outputs of logic gates together!



Thank You for Your Attention

- Read the lab assignment before you show up for your session!
- Continue reading the book
- Continue homework 1