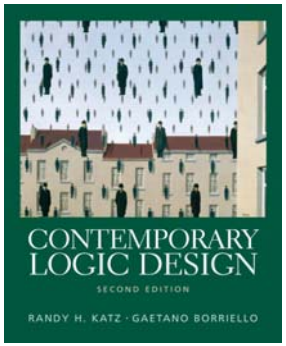
CSE 370 Spring 2006
Introduction to Digital Design

Lecture 19: More Moore and Mealy Machines



Last Lecture

- Moore and Mealy Machines

Today

- Moore and Mealy Machines

Administrivia

- HW 7 Out
- Lab 7 this week
- Quiz 3 graded, pick up (Average 8.74, Median 9.9)

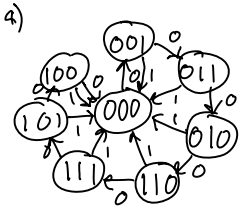
Quiz Review

1. In this problem we want to design a three bit Grey code counter. This counter will cycle through the following states
   000,001,011,010,110,111,101,100,000,etc.
   in the order listed above. Further the counter should provide a reset input (call it R) which if it is set will return the counter to the state 000.

a) Draw a state transition diagram for this counter. Each state should be labeled by the three bits listed above. You should label the transitions between these states with arrows labeled by the value R takes.

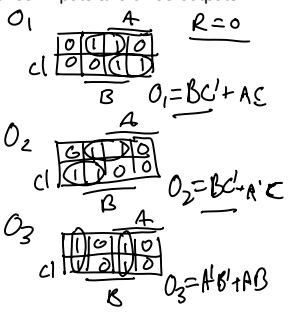
b) Create and fill out a state transition table for this counter. It should have four inputs and three outputs.

a)



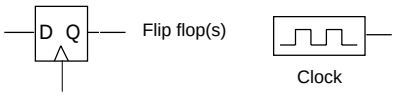
b)

R A B C	O_1 O_2 O_3
0 0 0 0	0 0 1
0 0 0 1	0 1 1
0 0 1 0	1 1 0
0 0 1 1	0 1 0
0 1 0 0	0 0 0
0 1 0 1	1 0 0
0 1 1 0	1 1 1
0 1 1 1	1 0 1
1 - - -	0 0 0



Quiz Review

c) Draw a circuit which implements the counter and changes state at the positive edge of the clock. Your circuit should have as input R and as output the bits of the counter, S1, S2, and S3. You are allowed to use any-fan in AND and OR gates, inverters as well as

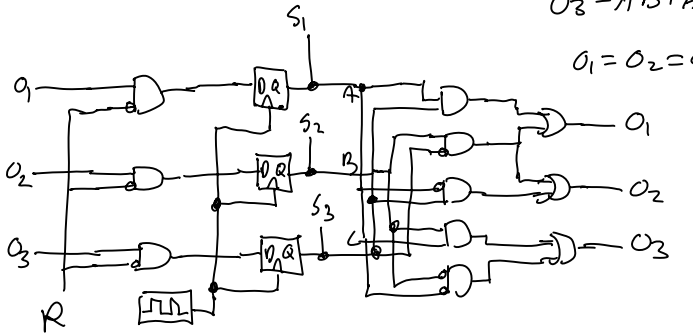


$O_1 = BC' + AC$ $\quad R=0$

$O_2 = BC' + A'C$

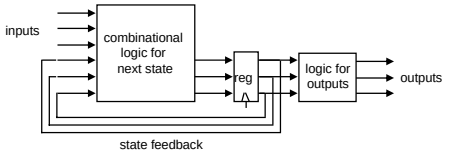
$O_3 = AB + A'B'$ $\quad R=1$

$O_1 = O_2 = O_3 = 0$

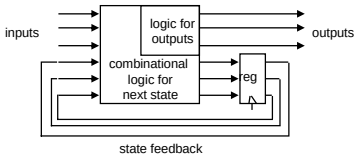


Comparison of Mealy and Moore machines (cont'd)

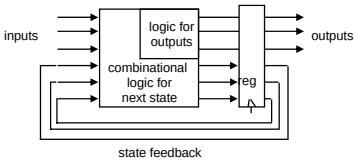
- Moore



- Mealy



- Synchronous Mealy



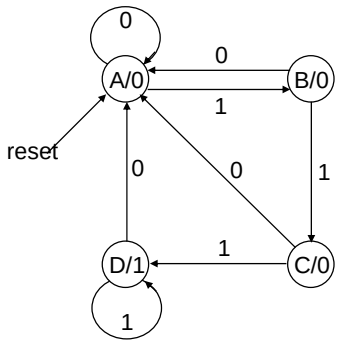
FSM design

- FSM-design procedure
  1. State diagram and state-transition table
  2. State minimization
  3. State assignment (or state encoding)
  4. Minimize next-state logic
  5. Implement the design

Example: Sequence detector

- Design a circuit to detect 3 or more 1's in a bit string
  - Assume Moore machine
  - Assume D flip-flops
  - Assume flip-flops have a reset

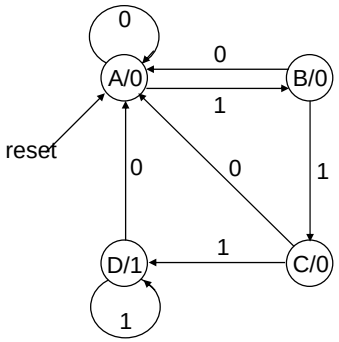
1. State diagram and state-transition table



reset	current state	input	next state	current output
1	–	–	A	0
0	A	0	A	0
0	A	1	B	0
0	B	0	A	0
0	B	1	C	0
0	C	0	A	0
0	C	1	D	0
0	D	0	A	1
0	D	1	D	1

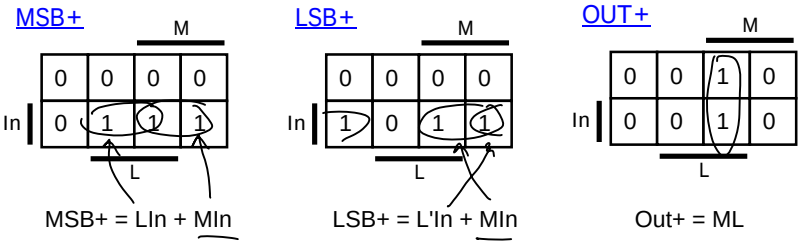
2. State minimization & 3. State encoding

- State diagram is already minimized
- Try a binary encoding



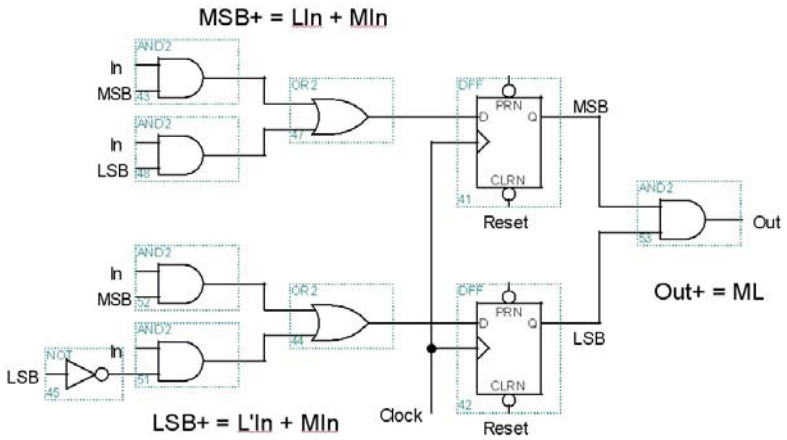
reset	current state	input	next state	current output
1	–	–	00	0
0	00	0	00	0
0	00	1	01	0
0	01	0	00	0
0	01	1	10	0
0	10	0	00	0
0	10	1	11	0
0	11	0	00	1
0	11	1	11	1

4. Minimize next-state logic



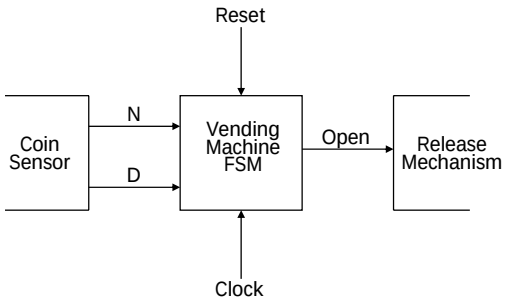
Notation
M := MSB
L := LSB
In := Input

5. Implement the design



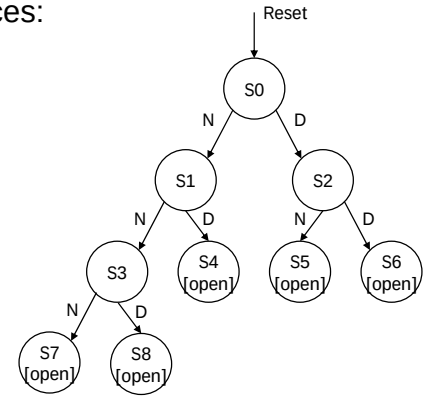
Example: vending machine

- Release item after 15 cents are deposited
- Single coin slot for dimes, nickels
- No change



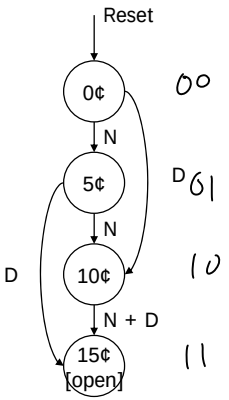
Example: vending machine (cont'd)

- Suitable abstract representation
  - tabulate typical input sequences:
    - 3 nickels
    - nickel, dime
    - dime, nickel
    - two dimes
  - draw state diagram:
    - inputs: N, D, reset
    - output: open chute
  - assumptions:
    - assume N and D asserted for one cycle
    - each state has a self loop for N = D = 0 (no coin)



Example: vending machine (cont'd)

- Minimize number of states - reuse states whenever possible



present state	inputs D	N	next state	output open
0¢	0	0	0¢	0
	0	1	5¢	0
	1	0	10¢	0
	1	1	–	–
5¢	0	0	5¢	0
	0	1	10¢	0
	1	0	15¢	0
	1	1	–	–
10¢	0	0	10¢	0
	0	1	15¢	0
	1	0	15¢	0
	1	1	–	–
15¢	–	–	15¢	1

symbolic state table

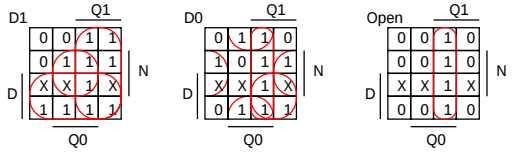
Example: vending machine (cont'd)

- Uniquely encode states

present state Q1	Q0	inputs D	N	next state D1	D0	output open
0	0	0	0	0	0	0
		0	1	0	1	0
		1	0	1	0	0
		1	1	–	–	–
0	1	0	0	0	1	0
		0	1	1	0	0
		1	0	1	1	0
		1	1	–	–	–
1	0	0	0	1	0	0
		0	1	1	1	0
		1	0	1	1	0
		1	1	–	–	–
1	1	–	–	1	1	1

Example: Moore implementation

- Mapping to logic



D1 = Q1 + D + Q0 N

D0 = Q0' N + Q0 N' + Q1 N + Q1 D

OPEN = Q1 Q0

Example: vending machine (cont'd)

- One-hot encoding

present state Q3 Q2 Q1 Q0	inputs D N	next state output D3 D2 D1 D0	open
0 0 0 1	0 0	0 0 0 1	0
	0 1	0 0 1 0	0
	1 0	0 1 0 0	0
	1 1	- - - -	-
0 0 1 0	0 0	0 0 1 0	0
	0 1	0 1 0 0	0
	1 0	1 0 0 0	0
	1 1	- - - -	-
0 1 0 0	0 0	0 1 0 0	0
	0 1	1 0 0 0	0
	1 0	1 0 0 0	0
	1 1	- - - -	-
1 0 0 0	- -	1 0 0 0	1

D0 = Q0 D' N'

D1 = Q0 N + Q1 D' N'

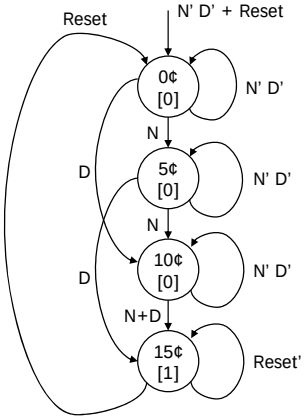
D2 = Q0 D + Q1 N + Q2 D' N'

D3 = Q1 D + Q2 D + Q2 N + Q3

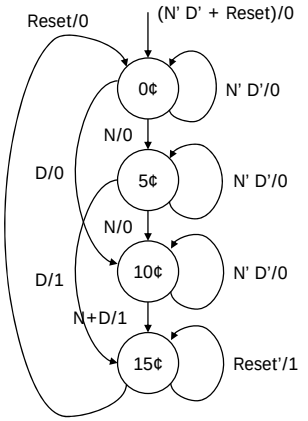
OPEN = Q3

Equivalent Mealy and Moore state diagrams

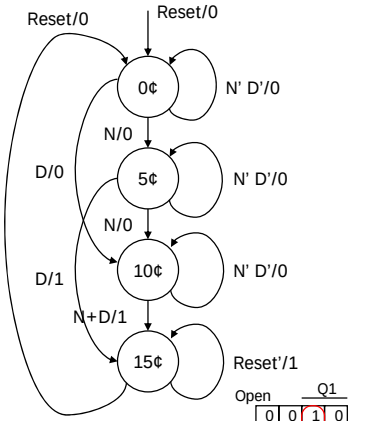
- Moore machine
  - outputs associated with state



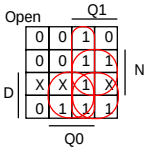
- Mealy machine
  - outputs associated with transitions



Example: Mealy implementation



present state Q1 Q0	inputs D N	next state D1 D0	output open
0 0	0 0	0 0	0
	0 1	0 1	0
	1 0	1 0	0
	1 1	– –	–
0 1	0 0	0 1	0
	0 1	1 0	0
	1 0	1 1	1
	1 1	– –	–
1 0	0 0	1 0	0
	0 1	1 1	1
	1 0	1 1	1
	1 1	– –	–
1 1	– –	1 1	1



D0 = Q0'N + Q0N' + Q1N + Q1D

D1 = Q1 + D + Q0N

OPEN = Q1Q0 + Q1N + Q1D + Q0D

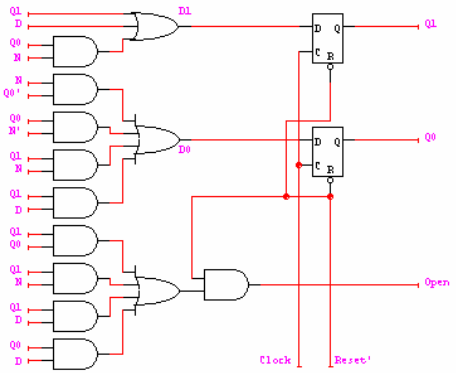
Example: Mealy implementation

D0 = Q0'N + Q0N' + Q1N + Q1D

D1 = Q1 + D + Q0N

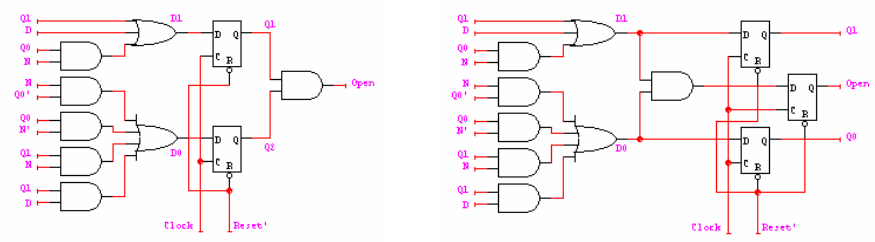
OPEN = Q1Q0 + Q1N + Q1D + Q0D

make sure OPEN is 0 when reset – by adding AND gate



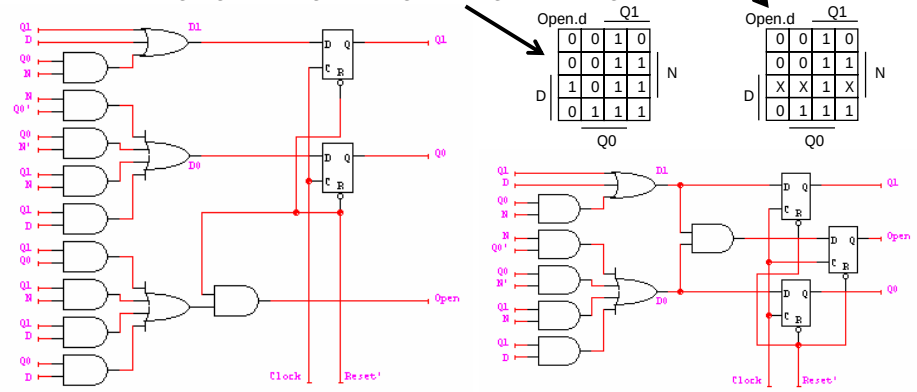
Vending machine: Moore to synch. Mealy

- OPEN = Q1Q0 creates a combinational delay after Q1 and Q0 change in Moore implementation
- This can be corrected by retiming, i.e., move flip-flops and logic through each other to improve delay
- OPEN.d = (Q1 + D + Q0N)(Q0'N + Q0N' + Q1N + Q1D)
  = Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D
- Implementation now looks like a synchronous Mealy machine
  - it is common for programmable devices to have FF at end of logic



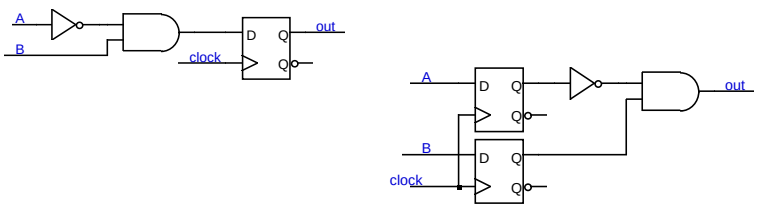
Vending machine: Mealy to synch. Mealy

- OPEN.d = Q1Q0 + Q1N + Q1D + Q0D
- OPEN.d = (Q1 + D + Q0N)(Q0'N + Q0N' + Q1N + Q1D)
  = Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D



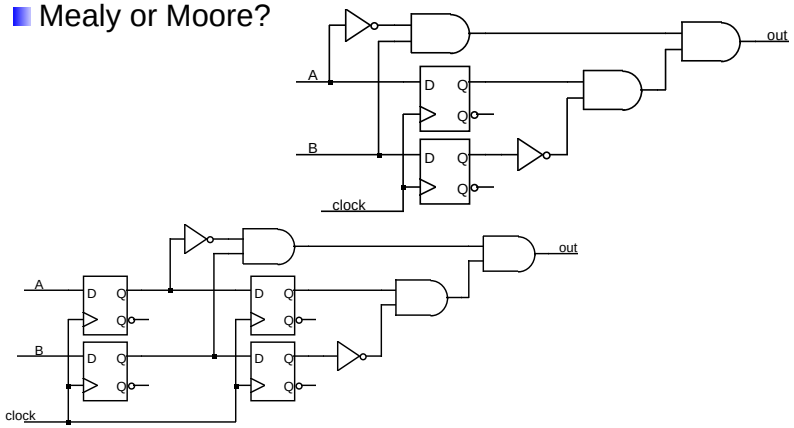
Mealy and Moore examples

- Recognize A,B = 0,1
  - Mealy or Moore?



Mealy and Moore examples (cont'd)

- Recognize A,B = 1,0 then 0,1
  - Mealy or Moore?

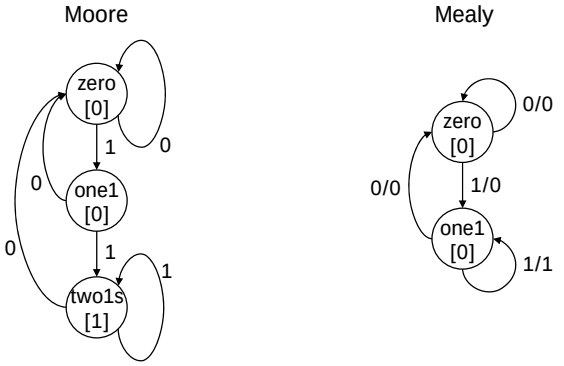


HDLs and Sequential Logic

- Flip-flops
  - representation of clocks - timing of state changes
  - asynchronous vs. synchronous
- FSMs
  - structural view (FFs separate from combinational logic)
  - behavioral view (synthesis of sequencers – not in this course)
- Data-paths = data computation (e.g., ALUs, comparators) + registers
  - use of arithmetic/logical operators
  - control of storage elements

Example: reduce-1-string-by-1

- Remove one 1 from every string of 1s on the input

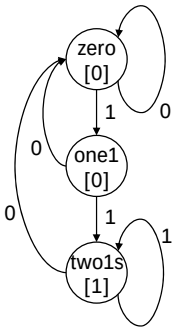


Verilog FSM - Reduce 1s example

- Moore machine

```
module reduce (clk, reset, in, out);
  input clk, reset, in;
  output out;

  parameter zero  = 2'b00;
  parameter one1  = 2'b01;
  parameter two1s = 2'b10;

  reg out;
  reg [2:1] state;       // state variables
  reg [2:1] next_state;

  always @(posedge clk)
    if (reset) state = zero;
    else       state = next_state;
```

state assignment (easy to change, if in one place)



Moore Verilog FSM (cont'd)

```
always @(in or state)

  case (state)
    zero:
   // last input was a zero
    begin
      if (in) next_state = one1;
      else    next_state = zero;
    end
     one1:
   // we've seen one 1
    begin
      if (in) next_state = two1s;
      else    next_state = zero;
    end
     two1s:
   // we've seen at least 2 ones
    begin
      if (in) next_state = two1s;
      else    next_state = zero;
    end
  endcase
```

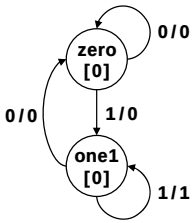
crucial to include all signals that are input to state determination

note that output depends only on state

```
always @(state)
  case (state)
    zero: out = 0;
    one1: out = 0;
   two1s: out = 1;
  endcase

endmodule
```

Mealy Verilog FSM

```
module reduce (clk, reset, in, out);
  input clk, reset, in;
  output out;
  reg out;
  reg state; // state variables
  reg next_state;

  always @(posedge clk)
    if (reset) state = zero;
    else       state = next_state;

  always @(in or state)
    case (state)
      zero:             // last input was a zero
     begin
       out = 0;
       if (in) next_state = one;
       else    next_state = zero;
     end
      one:              // we've seen one 1
     if (in) begin
        next_state = one; out = 1;
     end else begin
        next_state = zero; out = 0;
     end
    endcase
endmodule
```



Synchronous Mealy Machine

```
module reduce (clk, reset, in, out);
  input clk, reset, in;
  output out;
  reg out;
  reg state; // state variables

  always @(posedge clk)
    if (reset) state = zero;
    else
     case (state)
      zero:      // last input was a zero
     begin
       out = 0;
       if (in) state = one;
       else    state = zero;
     end
      one:       // we've seen one 1
     if (in) begin
        state = one; out = 1;
     end else begin
        state = zero; out = 0;
     end
    endcase
endmodule
```

Finite state machines summary

- Models for representing sequential circuits
  - abstraction of sequential elements
  - finite state machines and their state diagrams
  - inputs/outputs
  - Mealy, Moore, and synchronous Mealy machines
- Finite state machine design procedure
  - deriving state diagram
  - deriving state transition table
  - determining next state and output functions
  - implementing combinational logic
- Hardware description languages