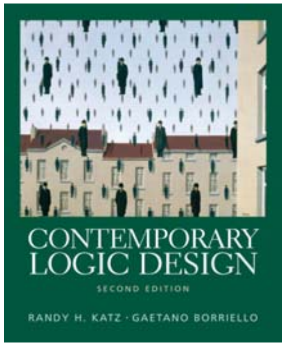
CSE 370 Spring 2006
Introduction to Digital Design

Lecture 6: Karnaugh Maps



Last Lecture
- Canonical Forms
- Sum~~s~~ of Products
- Product of Sums
- Boolean Cubes

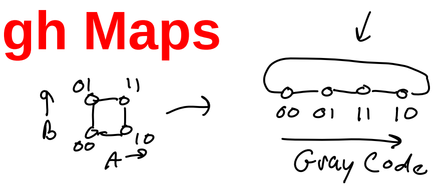
Today
- Karnaugh Maps

Administrivia

- Turn in Homework #2.
- Homework #3 available this afternoon on website.
- Office Hours: Firat Kiyak, Th 10-12am, in CSE 003
- Lab 3 available on website.
- Reading: Reading: pp. 93-114, 139-145, Verilog Reference (on website, see master calendar)

QUIZ #1

Karnaugh Maps



- Flat map of Boolean cube
  - wrap–around at edges
  - hard to draw and visualize for more than 4 dimensions
  - virtually impossible for more than 6 dimensions
- Alternative to truth-tables to help visualize adjacencies
  - guide to applying the uniting theorem
  - on-set elements with only one variable changing value are adjacent unlike the situation in a linear truth-table

A=B=0

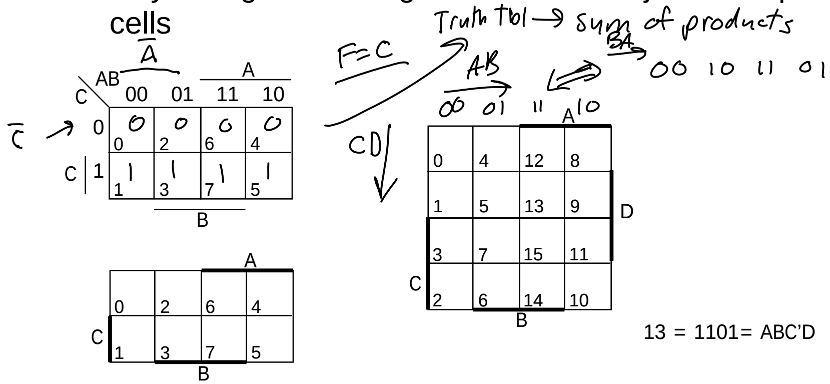
B \ A	0	1
0	1 (0)	1 (2)
1	0 (1)	0 (3)

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

$F = B'$

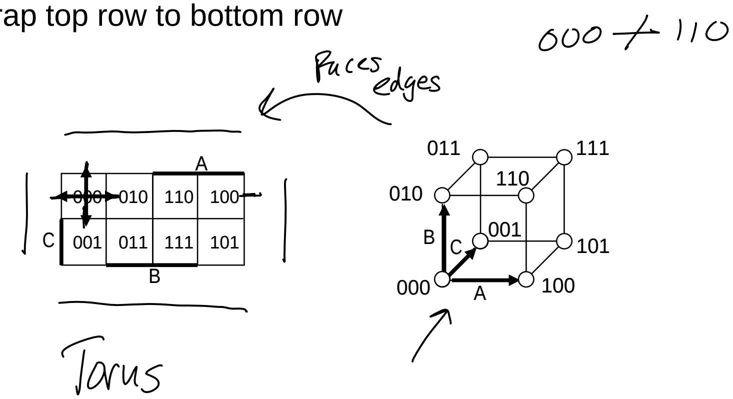
Karnaugh Maps Continued

- Numbering scheme based on Gray–code
  - e.g., 00, 01, 11, 10
  - only a single bit changes in code for adjacent map cells



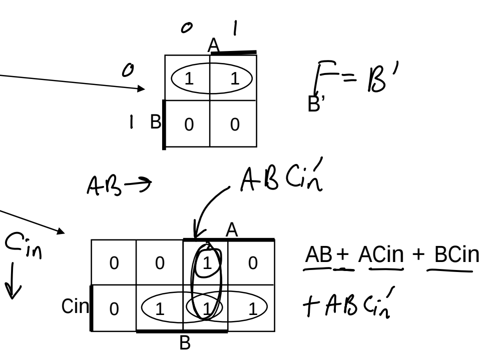
Adjacencies in Karnaugh Maps

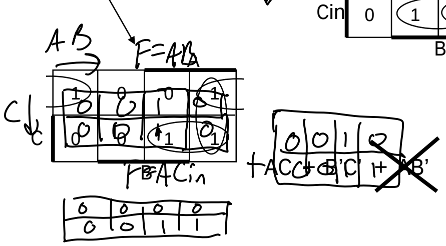
- Wrap from first to last column
- Wrap top row to bottom row



Karnaugh Map Examples

- F =
- Cout = 3 literals
- f(A,B,C) = Σm(0,4,5,7)

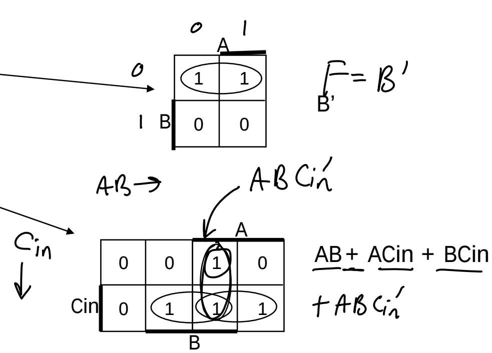


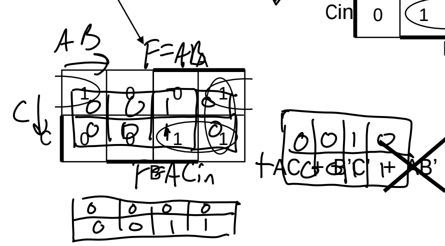


obtain the complement of the function by covering 0s with subcubes

Karnaugh Map Examples

- F =
- Cout = 3 literals
- f(A,B,C) = Σm(0,4,5,7)





obtain the complement of the function by covering 0s with subcubes

More Karnaugh Map Examples

			A	A
	0	0	1	1
C	0	0	1	1
		B	B	

G(A,B,C) = A

C'

			A	A
	1	0	0	1
C	0	0	1	1
		B	B	

F(A,B,C) = Σm(0,4,5,7) = AC + B'C'

			A	A
	0	1	1	0
C	1	1	0	0
		B	B	

F' simply replace 1's with 0's and vice versa

F'(A,B,C) = Σ m(1,2,3,6) = BC' + A'C

A Four Variable Example

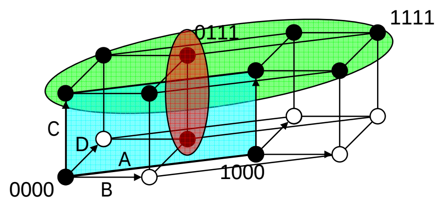
- F(A,B,C,D) = Σm(0,2,3,5,6,7,8,10,11,14,15) canonical

F = C + A'BD + B'D' DBA'C'

			A	A	
	1	0	0	1	
	0	1	0	0	D
C	1	1	1	1	D
C	1	1	1	1	
		B	B		

D'BA'C

A'BD(C+C')



find the smallest number of the largest possible subcubes to cover the ON-set
(fewer terms with fewer inputs per term)

Karnaugh Map Don't Cares

- f(A,B,C,D) = Σ m(1,3,5,7,9) + d(6,12,13)
  - without don't cares
    - f = A'D + B'C'D

AB

CD	00	01	11 (A)	10 (A)	
00	0	0	X	0	
01	1	1	X	1	D
11 (C)	1	1	0	0	D
10 (C)	0	X	0	0	
		B	B		

0111 | 1 → A'BCD

BA'CD

Karnaugh Map Don't Cares

- f(A,B,C,D) = Σ m(1,3,5,7,9) + d(6,12,13)
  - f = A'D + B'C'D without don't cares
  - f = A'D + C'D with don't cares

			A	A	
	0	0	X	0	
	1	1	X	1	D
C	1	1	0	0	D
C	0	X	0	0	
		B	B		

by using don't care as a "1"
a 2-cube can be formed
rather than a 1-cube to cover
this node

<u>don't cares</u> can be treated as
1s or 0s
depending on which is more
advantageous

Exercise

- Minimize the function $F = \Sigma\, m(0, 2, 7, 8, 14, 15) + d(3, 6, 9, 12, 13)$