#### Overview

- Optimizing FSMs
  - **∠** Pipelining
  - **∠**Retiming
  - **∠** Partitioning
- Conclusion of sequential logic

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#### **Definitions**

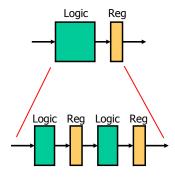
- Latency: Time to perform a computation
  - Data input to data output
- ◆ Throughput: Input or output data rate
  - Typically the clock rate
- Combinational delays drive performance
  - $d \equiv delay through slowest combinational stage$  $n \equiv$  number of stages from input to output

  - Throughput 

    1/d (in Hz)

# **Pipelining**

- What?
  - Subdivide combinational logic
  - Add registers between logic
- Why?
  - Trade latency for throughput
  - Increased throughput
    - ✔ Reduce logic delays
    - **∠** Increase clock speed
  - Increased latency
    - ✓ Takes cycles to fill the pipe
  - Increase circuit utilization
    - **∠** Simultaneous computations

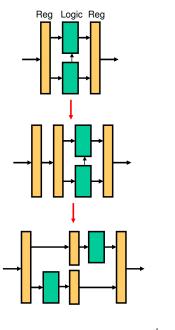


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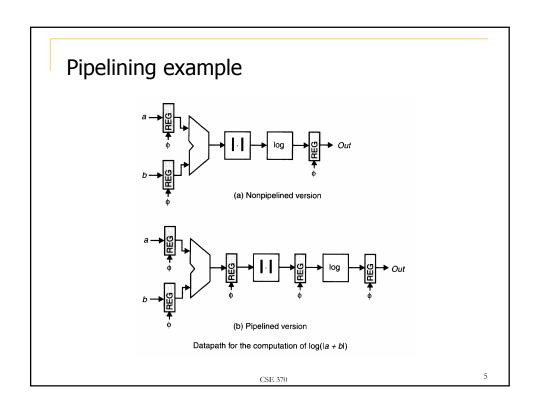
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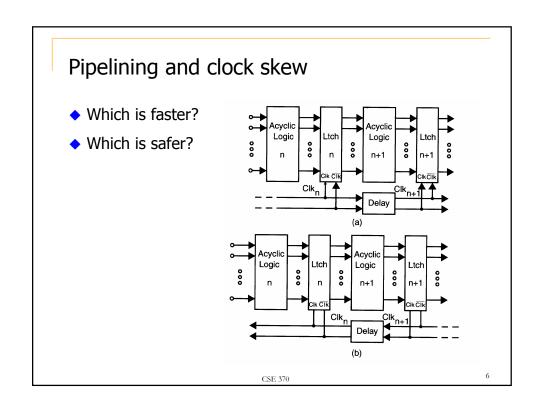
# **Pipelining**

- When?
  - Need throughput more than latency 
    ✓ Signal processing
  - Logic delays > setup/hold times
  - Acyclic logic
- Where?
  - At natural breaks in the combinational logic
  - Adding registers makes sense



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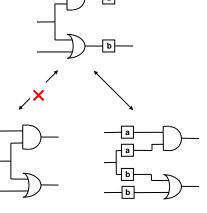
# Retiming

- Pipelining adds registers
  - To increase the clock speed
- Retiming moves registers around
  - Reschedules computations to optimize performance
    - ✓ Minimize critical path
    - ∠ Optimize logic across register boundaries
    - **∠** Reduce register count
  - Without altering functionality

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# Retiming in a nutshell

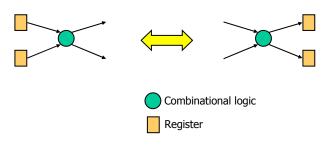
- Change position of FFs
  - For speed
  - To suit implementation target
- Retiming modifies state assignment
  - Preserves FSM functionality



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# Retiming groundrules

- Rules:
  - Remove one register from each input and add one to each output
  - Remove one register from each output and add one to each input



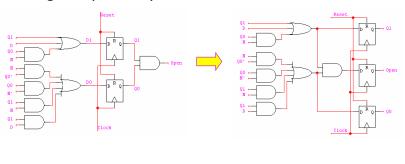
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# Retiming examples

Reduce register count



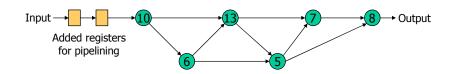
Change output delays

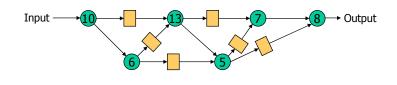


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# Optimal pipelining

- Add registers
- Use retiming to optimize location

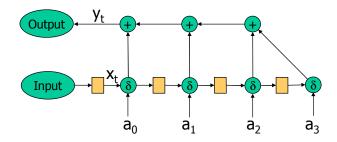




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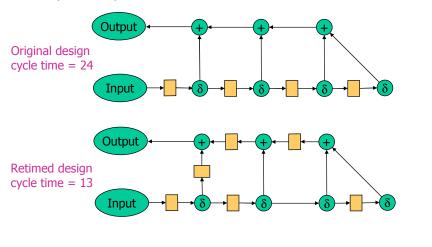
# Example: Digital correlator

- $y_t = \delta(x_t, a_0) + \delta(x_{t-1}, a_1) + \delta(x_{t-2}, a_2) + \delta(x_{t-3}, a_3)$ 
  - $\delta(x, a) = 1$  if x = a; 0 otherwise



# Example: Digital correlator (cont'd)

◆ Delays: Comparator = 3; adder = 7



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# FSM partitioning

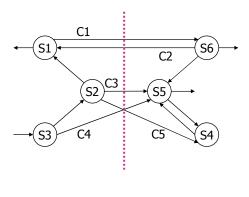
- ◆ Break a large FSM into two or more smaller FSMs
- Rationale
  - Less states in each partition
    - **∠** Simpler minimization and state assignment

    - ✓ Shorter critical path
  - But more logic overall
- Goal
  - Minimize communication between partitions
    - ✓ Minimize wires & I/O
- Partitions are synchronous
  - Same clock!!!

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# Example: Partition the machine

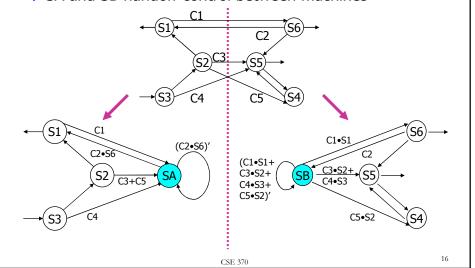
Partition into two halves



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### Introduce idle states

◆ SA and SB handoff control between machines



# Partitioning rules

Rule #1: Source-state transformation Replace by transition to idle state (SA)



Rule #2: Destination state transformation Replace with exit transition from idle state



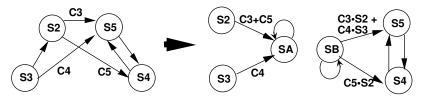
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# Partitioning rules (con't)

Rule #3: Multiple transitions with same source or destination Source ⇒ Replace by transitions to idle state (SA)

Destination ⇒ Replace with exit transitions from idle state



Rule #4: Hold condition for idle state OR exit conditions and invert



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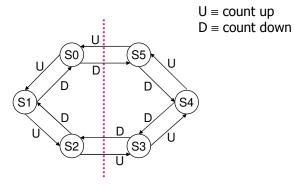
### Mealy versus Moore partitions

- Mealy machines undesirable
  - Inputs can affect outputs immediately w`output" can be a handoff to another machine!!!
  - Inputs can ripple through several machines in one clock cycle
- Moore or synchronized Mealy desirable
  - Input-to-output path always broken by a flip-flop
  - But...may take several clocks for input to propagate to output ✔ Output may derive from other side of a partition

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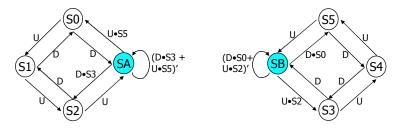
# Example: Six-state up/down counter

Break into 2 parts



#### Example: 6 state up/down counter (con't)

- ◆ Count sequence S<sub>0</sub>, S<sub>1</sub>, S<sub>2</sub>, S<sub>3</sub>, S<sub>4</sub>, S<sub>5</sub>
  - S<sub>2</sub> goes to S<sub>A</sub> and holds, leaves after S<sub>5</sub>
  - S<sub>5</sub> goes to S<sub>B</sub> and holds, leaves after S<sub>2</sub>
  - Down sequence is similar



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#### Minimize communication between partitions

- Ideal world: Two machines handoff control
  - Separate I/O, states, etc.
- Real world: Minimize handoffs and common I/O
  - Minimize number of state bits that cross boundary
  - Merge common outputs
- Look for:
  - Disjoint inputs used in different regions of state diagram
  - Outputs active in only one region of state diagram
  - Isomorphic portions of state diagram

    ✓ Add states, if necessary, to make them so
  - Regions of diagram with a single entry and single exit point

# Sequential logic: What you should know

- Sequential logic building blocks
  - Latches (R-S and D)
  - Flip-flops (master/slave D, edge-triggered D & T)
  - Latch and flip-flop timing (setup/hold time, prop delay)
  - Timing diagrams
  - Flip-flop clocking
  - Asynchronous inputs and metastability
  - Registers

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# Sequential logic: What you should know

- Counters
  - Timing diagrams
  - Shift registers
  - Ripple counters
  - State diagrams and state-transition tables
  - Counter design procedure
    - 1. Draw a state diagram
    - 2. Draw a state-transition table
    - 3. Encode the next-state functions
    - 4. Implement the design
  - Self-starting counters

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# Sequential logic: What you should know

- Finite state machines
  - Timing diagrams (synchronous FSMs)
  - Moore versus Mealy versus registered Mealy
  - FSM design procedure
    - 1. Understand the problem (state diagram & state-transition table)
    - 2. Determine the machine's states (minimize the state diagram)
    - 3. Encode the machine's states (state assignment)
    - 4. Design the next-state logic (minimize the combinational logic)
    - 5. Implement the FSM
  - FSM design guidelines
    - ✓ Separate datapath and control
  - One-hot encoding
  - Pipelining and retiming basics