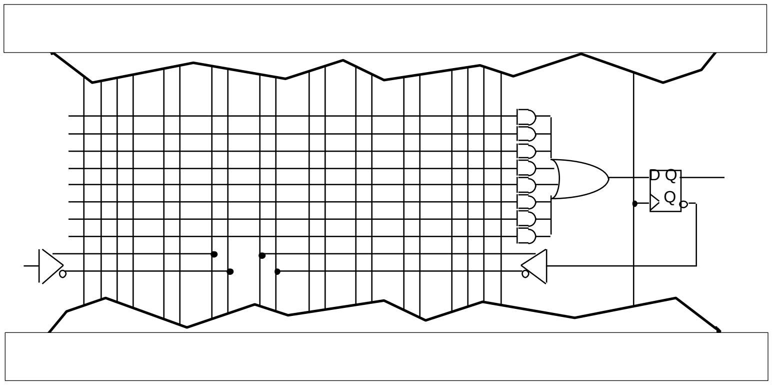
Sequential logic implementation

- Finite-state machines
  - Moore
  - Mealy
  - Synchronous Mealy
- Implementation
  - random logic gates and FFs
  - programmable logic devices (PAL with FFs)
- Design procedure
  - state diagrams
  - state transition table
  - state assignment
  - next state functions

Implementation using PALs

- Programmable logic building block for sequential logic
  - macro-cell: FF + logic
    - D-FF
    - two-level logic capability like PAL (e.g., 8 product terms)

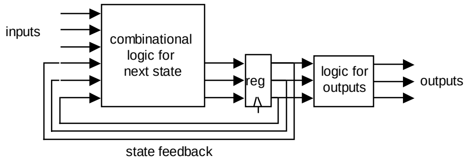


Comparison of Mealy and Moore machines

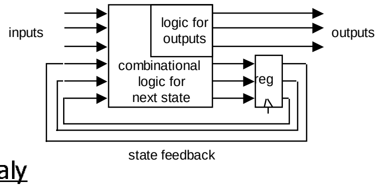
- Mealy machines tend to have less states
  - different outputs on arcs (n^2) rather than states (n)
- Moore machines are safer to use
  - outputs change at clock edge (always one cycle later)
  - in Mealy machines, input change can cause output change as soon as logic is done – a big problem when two machines are interconnected – asynchronous feedback
- Mealy machines react faster to inputs
  - react in same cycle – don't need to wait for clock
  - in Moore machines, more logic may be necessary to decode state into outputs – more gate delays after

Comparison of Mealy and Moore machines (cont'd)

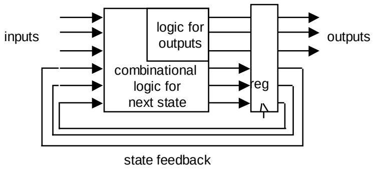
- Moore



- Mealy

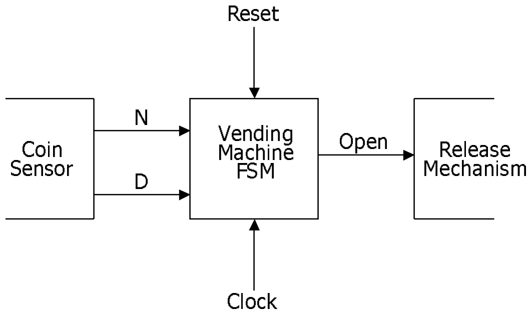


- Synchronous Mealy



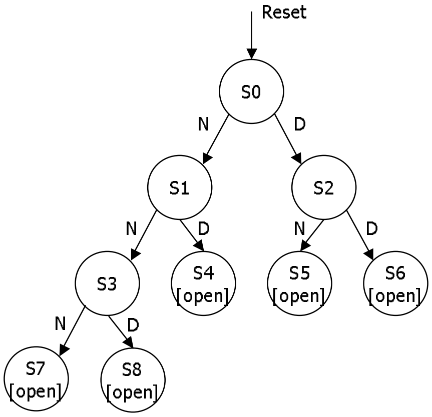
Example: vending machine

- Release item after 15 cents are deposited
- Single coin slot for dimes, nickels
- No change



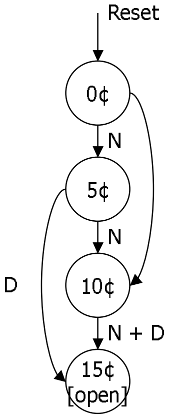
Example: vending machine (cont'd)

- Suitable abstract representation
  - tabulate typical input sequences:
    - 3 nickels
    - nickel, dime
    - dime, nickel
    - two dimes
  - draw state diagram:
    - inputs: N, D, reset
    - output: open chute
  - assumptions:
    - assume N and D asserted for one cycle
    - each state has a self loop for N = D = 0 (no coin)



Example: vending machine (cont'd)

- Minimize number of states - reuse states whenever possible



present state	inputs D	N	next state	output open
0¢	0	0	0¢	0
	0	1	5¢	0
	1	0	10¢	0
	1	1	–	–
5¢	0	0	5¢	0
	0	1	10¢	0
	1	0	15¢	0
	1	1	–	–
10¢	0	0	10¢	0
	0	1	15¢	0
	1	0	15¢	0
	1	1	–	–
15¢	–	–	15¢	1

symbolic state table

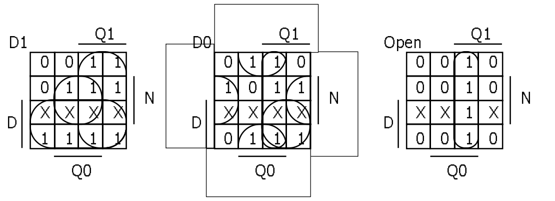
Example: vending machine (cont'd)

- Uniquely encode states

present state Q1	Q0	inputs D	N	next state D1	D0	output open
0	0	0	0	0	0	0
		0	1	0	1	0
		1	0	1	0	0
		1	1	–	–	–
0	1	0	0	0	1	0
		0	1	1	0	0
		1	0	1	1	0
		1	1	–	–	–
1	0	0	0	1	0	0
		0	1	1	1	0
		1	0	1	1	0
		1	1	–	–	–
1	1	–	–	1	1	1

Example: Moore implementation

- Mapping to logic



D1

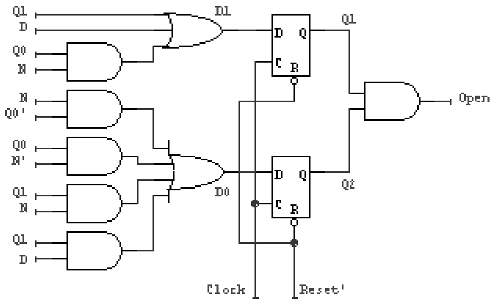
0	0	1	1
0	1	1	1
X	X	X	X
1	1	1	1

D0

0	1	1	0
1	0	1	1
X	X	X	X
0	1	1	1

Open

0	0	1	0
0	0	1	0
X	X	1	X
0	0	1	0



D1 = Q1 + D + Q0 N

D0 = Q0′ N + Q0 N′ + Q1 N + Q1 D

OPEN = Q1 Q0

Example: vending machine (cont'd)

- One-hot encoding

present state				inputs		next state				output
Q3	Q2	Q1	Q0	D	N	D3	D2	D1	D0	open
0	0	0	1	0	0	0	0	0	1	0
				0	1	0	0	1	0	0
				1	0	0	1	0	0	0
				1	1	-	-	-	-	-
0	0	1	0	0	0	0	0	1	0	0
				0	1	0	1	0	0	0
				1	0	1	0	0	0	0
				1	1	-	-	-	-	-
0	1	0	0	0	0	0	1	0	0	0
				0	1	1	0	0	0	0
				1	0	1	0	0	0	0
				1	1	-	-	-	-	-
1	0	0	0	-	-	1	0	0	0	1

D0 = Q0 D′ N′

D1 = Q0 N + Q1 D′ N′

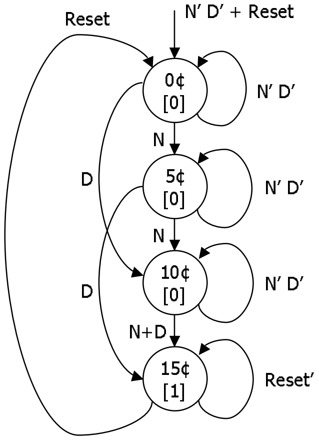
D2 = Q0 D + Q1 N + Q2 D′ N′

D3 = Q1 D + Q2 D + Q2 N + Q3

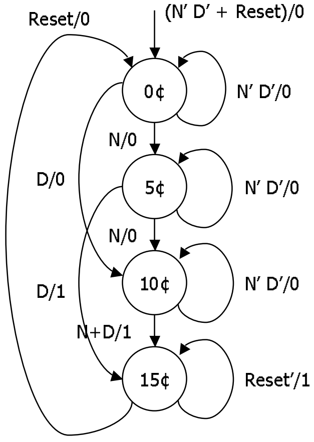
OPEN = Q3

Equivalent Mealy and Moore state diagrams

- Moore machine
  - outputs associated with state

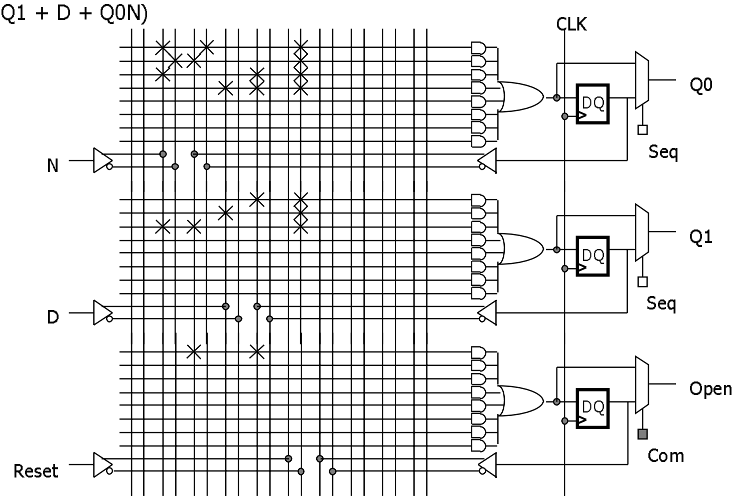


- Mealy machine
  - outputs associated with transitions

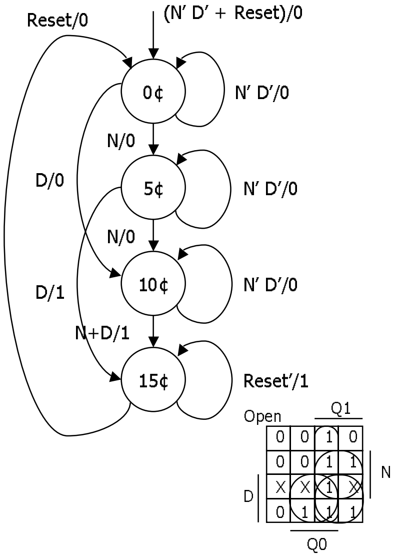


Vending machine example (Moore PLD mapping)

D0 = reset'(Q0'N + Q0N' + Q1N + Q1D)
D1 = reset'(Q1 + D + Q0N)
OPEN = Q1Q0



Example: Mealy implementation



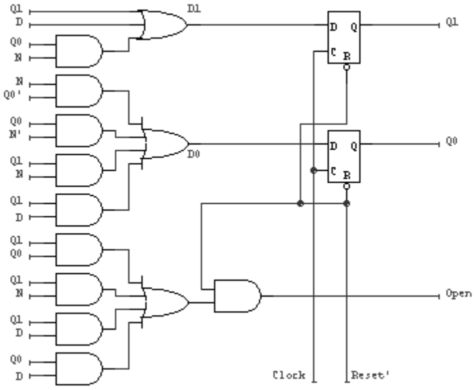
0	0	1	0			
0	0	1	1			
X	X	1	X			
0	1	1	1			

present state		inputs		next state		output
Q1	Q0	D	N	D1	D0	open
0	0	0	0	0	0	0
		0	1	0	1	0
		1	0	1	0	0
		1	1	–	–	–
0	1	0	0	0	1	0
		0	1	1	0	0
		1	0	1	1	1
		1	1	–	–	–
1	0	0	0	1	0	0
		0	1	1	1	1
		1	0	1	1	1
		1	1	–	–	–
1	1	–	–	1	1	1

D0 = reset'(Q0'N + Q0N' + Q1N + Q1D)
D1 = reset'(Q1 + D + Q0N)
OPEN = reset'(Q1Q0 + Q1N + Q1D + Q0D)

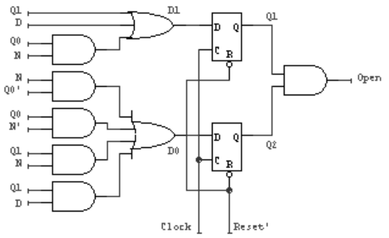
Example: Mealy implementation

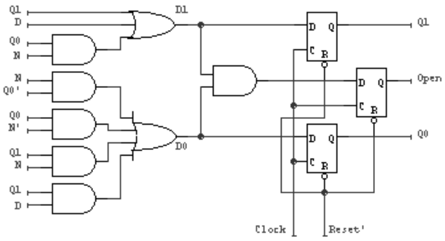
D0 = reset'(Q0'N + Q0N' + Q1N + Q1D)
D1 = reset'(Q1 + D + Q0N)
OPEN = reset'(Q1Q0 + Q1N + Q1D + Q0D)



Vending machine: Moore to synch. Mealy

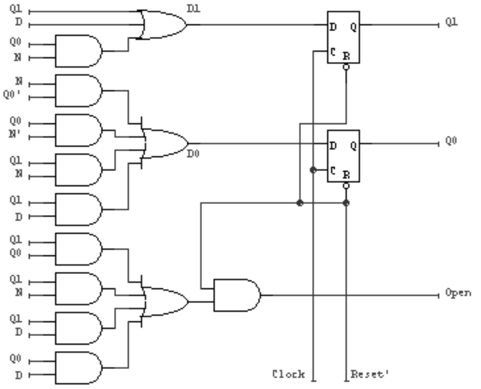
- OPEN= Q1Q0 creates a combinational delay after Q1 and Q0 change in Moore implementation
- This can be corrected by retiming, i.e., move flip-flops and logic through each other to improve delay
- OPEN= reset'(Q1 + D + Q0N)(Q0'N + Q0N' + Q1N + Q1D)
  = reset'(Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D)
- Implementation now looks like a synchronous Mealy machine
  - it is common for programmable devices to have FF at end of logic





Vending machine: Mealy to synch. Mealy

- OPEN= reset'(Q1Q0 + Q1N + Q1D + Q0D)
- OPEN= reset'(Q1 + D + Q0N)(Q0'N + Q0N' + Q1N + Q1D)
  = reset'(Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D)



Open Q1

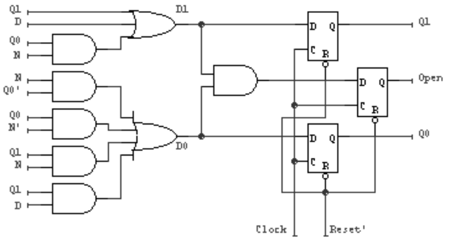
	0	0	1	0
	0	0	1	1
D	1	0	1	1
D	0	1	1	1

N, Q0

Open Q1

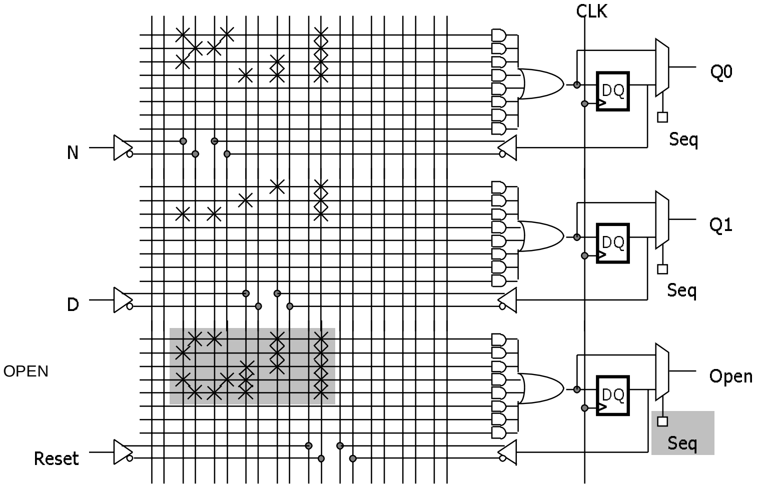
	0	0	1	0
	0	0	1	1
D	X	X	1	X
D	0	1	1	1

N, Q0



Vending machine (synch. Mealy PLD mapping)

OPEN = reset'(Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D)

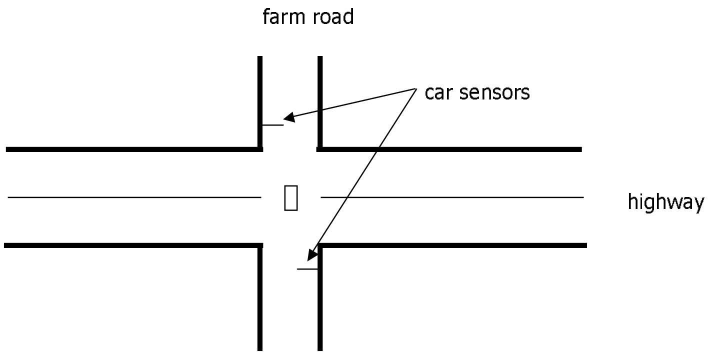


Example: traffic light controller

- A busy highway is intersected by a little used farmroad
- Detectors C sense the presence of cars waiting on the farmroad
  - with no car on farmroad, light remain green in highway direction
  - if vehicle on farmroad, highway lights go from Green to Yellow to Red, allowing the farmroad lights to become green
  - these stay green only as long as a farmroad car is detected but never longer than a set interval
  - when these are met, farm lights transition from Green to Yellow to Red, allowing highway to return to green
  - even if farmroad vehicles are waiting, highway gets at least a set interval as green
- Assume you have an interval timer that generates:
  - a short time pulse (TS) and
  - a long time pulse (TL),
  - in response to a set (ST) signal.
  - TS is to be used for timing yellow lights and TL for green lights

Example: traffic light controller (cont')

- Highway/farm road intersection



Example: traffic light controller (cont')

- Tabulation of inputs and outputs

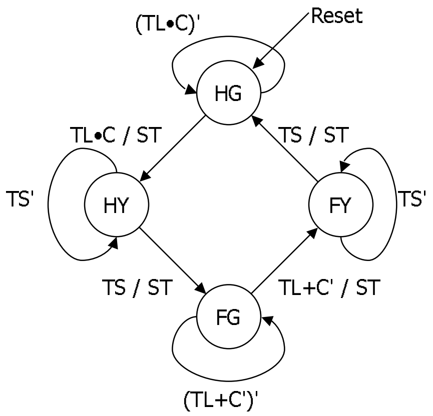
inputs	description	outputs	description
reset	place FSM in initial state	HG, HY, HR	assert green/yellow/red highway lights
C	detect vehicle on the farm road	FG, FY, FR	assert green/yellow/red highway lights
TS	short time interval expired	ST	start timing a short or long interval
TL	long time interval expired		

- Tabulation of unique states – some light configurations imply others

state	description
HG	highway green (farm road red)
HY	highway yellow (farm road red)
FG	farm road green (highway red)
FY	farm road yellow (highway red)

Example: traffic light controller (cont')

- State diagram



Example: traffic light controller (cont')

- Generate state table with symbolic states
- Consider state assignments

output encoding – similar problem
to state assignment
(Green = 00, Yellow = 01, Red = 10)

Inputs			Present State	Next State	Outputs		
C	TL	TS			ST	H	F
0	–	–	HG	HG	0	Green	Red
–	0	–	HG	HG	0	Green	Red
1	1	–	HG	HY	1	Green	Red
–	–	0	HY	HY	0	Yellow	Red
–	–	1	HY	FG	1	Yellow	Red
1	0	–	FG	FG	0	Red	Green
0	–	–	FG	FY	1	Red	Green
–	1	–	FG	FY	1	Red	Green
–	–	0	FY	FY	0	Red	Yellow
–	–	1	FY	HG	1	Red	Yellow

SA1:	HG = 00	HY = 01	FG = 11	FY = 10	
SA2:	HG = 00	HY = 10	FG = 01	FY = 11	
SA3:	HG = 0001	HY = 0010	FG = 0100	FY = 1000	(one-hot)

Logic for different state assignments

- SA1

  NS1 = C•TL'•PS1•PS0 + TS•PS1'•PS0 + TS•PS1•PS0' + C'•PS1•PS0 + TL•PS1•PS0
  NS0 = C•TL•PS1'•PS0' + C•TL'•PS1•PS0 + PS1'•PS0

  ST = C•TL•PS1'•PS0' + TS•PS1'•PS0 + TS•PS1•PS0' + C'•PS1•PS0 + TL•PS1•PS0
  H1 = PS1 H0 = PS1'•PS0
  F1 = PS1' F0 = PS1•PS0‘

- SA2

  NS1 = C•TL•PS1' + TS'•PS1 + C'•PS1'•PS0
  NS0 = TS•PS1•PS0' + PS1'•PS0 + TS'•PS1•PS0

  ST = C•TL•PS1' + C'•PS1'•PS0 + TS•PS1
  H1 = PS0 H0 = PS1•PS0'
  F1 = PS0' F0 = PS1•PS0

- SA3

  NS3 = C'•PS2 + TL•PS2 + TS'•PS3 NS2 = TS•PS1 + C•TL'•PS2
  NS1 = C•TL•PS0 + TS'•PS1 NS0 = C'•PS0 + TL'•PS0 + TS•PS3

  ST = C•TL•PS0 + TS•PS1 + C'•PS2 + TL•PS2 + TS•PS3
  H1 = PS3 + PS2 H0 = PS1
  F1 = PS1 + PS0 F0 = PS3

Sequential logic implementation summary

- Models for representing sequential circuits
  - finite state machines and their state diagrams
  - Mealy, Moore, and synchronous Mealy machines
- Finite state machine design procedure
  - deriving state diagram
  - deriving state transition table
  - assigning codes to states
  - determining next state and output functions
  - implementing combinational logic
- Implementation technologies
  - random logic + FFs
  - PAL with FFs (programmable logic devices – PLDs)