Quiz 1 CSE 370 Winter 2001

Sample Solutions

1) a) [5 pts] Prove the following equation using Boolean axioms and laws:

$$(x'\oplus y) = (x\oplus y)'$$

Indicate, at each step, which axiom/law you are applying.
Use this definition for $\oplus$: $x\oplus y = xy' + x'y$.

$(x\oplus y)'$	$= (xy'+x'y)'$	definition of $\oplus$
	$= (x'+y)*(x+y')$	generalized de Morgan's (15)
	$= x'y' + xy$	factoring (12)
	$= x'y' + x''y$	involution (4)
	$= (x'\oplus y)$	definition of $\oplus$

b) [5 pts] Consider the function defined by the following Karnaugh map. Find its optimal implementation using AND, OR, XOR and NOT gates with multiple inputs. Strive for the smallest number of gates.

x,y =	0,0	0,1	1,1	1,0
z = 0	1	0	1	0
z = 1	0	1	0	1

It is $(x\oplus y\oplus z)'$.

Can be implemented with a 3-input XOR gate and a NOT gate.

2) All subproblems of this problem refer to the same function.

a) [5 pts] For the function with the following Karnaugh map, write a minimal S-o-P form:

x,y =	0,0	0,1	1,1	1,0
z,w = 0,0	1	0	0	1
0,1	1	1	0	0
1,1	0	1	1	0
1,0	0	0	1	1

`For example, x'y'z' + x'yw + xyz + xy'w'.`

b) [5 pts] For the same function, write a minimal P-o-S form:

x,y =	0,0	0,1	1,1	1,0
z,w = 0,0	1	0	0	1
0,1	1	1	0	0
1,1	0	1	1	0
1,0	0	0	1	1

`For example, (x+y+z')*(x+y'+w)*(x'+y'+z)*(x'+y+w').`

c) [3 pts] How many prime implicants does this function have? Indicate all prime implicants on the Karnaugh map.

x,y =	0,0	0,1	1,1	1,0
z,w = 0,0	1	0	0	1
0,1	1	1	0	0
1,1	0	1	1	0
1,0	0	0	1	1

`There are 8 prime implicants.`

d) [3 pts] Indicate all *essential* prime implicants on the Karnaugh map.

This function has *no* essential prime implicants.

3) a) [7 pts] Find the S-o-P form of the following function using de Morgan's law:

$$f(X,Y,Z) = (X+Y'+Z) * (X'+Z')$$

First, create the P-o-S form of f' by making produce of all sums which are not in f:

```
f’  = (X+Y)*(X+Y’+Z’)*(X’+Y+Z)*(X’+Y’+Z)
```

Then, apply de Morgan's law:

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f   = X’Y’ + X’YZ + XY’Z’ + XYZ’
```

b) [2 pts] Using Boolean axioms and laws, simplify the obtained S-o-P form of f.

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f   = X’Y’ + X’YZ + XY’Z’ + XYZ’
    = X’Y’*(1+Z) + X’YZ + XZ’*(Y’+Y)
    = X’Y’ + X’Y’Z + X’YZ + XZ’
    = X’Y’ + X’Z + XZ’
```

4) [5 pts] For the following expression:

$$(W+X'+Y')*(W'+Z')*(W+Y)$$

obtain and simplify its S-o-P form using Boolean axioms and laws.

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      (W+X’+Y’)*(W’+Z’)*(W+Y)
    = (W+X’+Y’)*(W’Y+Z’W)
    = WW’Y + WWZ’ + W’X’Y + WX’Z’ + WYY’ + WY’Z’
    = WZ’ + W’X’Y + WX’Z’+WY’Z’
    = WZ’*(1+X’+Y’) + W’X’Y
    = WZ’ + W’X’Y
```

5) [5 pts] Specify a 4-variable function f(X,Y,Z,W) which has no essential prime implicants and, moreover, its complement, f', has no essential prime implicants either.

This is, for example, the function in Problem 2.