CSE341: Programming Languages

Lecture 10
References, Polymorphic Datatypes, the Value Restriction, Type Inference

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Callbacks

A common idiom: Library takes functions to apply later, when an *event* occurs – examples:

- When a key is pressed, mouse moves, data arrives
- When the program enters some state (e.g., turns in a game)

A library may accept multiple callbacks

- Different callbacks may need different private data with different types
- Fortunately, a function’s type does not include the types of bindings in its environment
- (In OOP, objects and private fields are used similarly, e.g., Java Swing’s event-listeners)

Mutable state

While it's not absolutely necessary, mutable state is reasonably appropriate here

- We really do want the "callbacks registered" and "events that have been delivered" to *change* due to function calls

For the reasons we have discussed, ML variables really are immutable, but there are mutable references (use sparingly)

- New types: **`t ref`** where **`t`** is a type
- New expressions:
  - **`ref e`** to create a reference with initial contents e
  - **`e1 := e2`** to update contents
  - **`!e`** to retrieve contents (not negation)

References example

```
val x = ref 42
val y = ref 42
val z = x
val _ = x := 43
val w = (!y) + (!z) (* 85 *)
(* x + 1 does not type-check)
```

- A variable bound to a reference (e.g., **`x`**) is still immutable: it will always refer to the same reference
- But the contents of the reference may change via **`:=`**
- And there may be aliases to the reference, which matter a lot
- Reference are first-class values
- Like a one-field mutable object, so **`:=`** and **`!`** don't specify the field

Example call-back library

Library maintains mutable state for "what callbacks are there" and provides a function for accepting new ones

- A real library would support removing them, etc.
- In example, callbacks have type **`int->unit`** (executed for side-effect)

So the entire public library interface would be the function for registering new callbacks:

```
val onKeyEvent : (int -> unit) -> unit
```

Library implementation

```
val cbs : (int -> unit) list ref = ref []

fun onKeyEvent f =  cbs := f :: (!cbs)

fun onEvent i =
    let fun loop fs =
            case fs of
              []     => ()
            | f::fs' => (f i; loop fs')
    in loop (!cbs) end
```

Clients

Can only register an **`int -> unit`**, so if any other data is needed, must be in closure's environment

– And if need to "remember" something, need mutable state

Examples:

```
val timesPressed = ref 0
val _ = onKeyEvent (fn _ =>
            timesPressed := (!timesPressed) + 1)

fun printIfPressed i =
   onKeyEvent (fn j =>
       if i=j
       then print ("pressed " ^ Int.toString i)
       else ())
```

More about types

- Polymorphic datatypes, type constructors
- Why do we need the Value Restriction?
- Type inference: behind the curtain

Polymorphic Datatypes

```
datatype int_list =
     EmptyList
   | Cons of int * int_list

datatype ‘a non_mt_list =
     One of ‘a
   | More of ‘a * (‘a non_mt_list)

datatype (‘a,‘b) tree =
     Leaf of ‘a
   | Node of ‘b * (‘a,‘b) tree * (‘a,‘b) tree

val t1 = Node(“hi”,Leaf 4,Leaf 8)
                  (* (int,string) tree *)
val t2 = Node(“hi”,Leaf true,Leaf 8)
                  (* does not typecheck *)
```

Polymorphic Datatypes

```
datatype 'a list = [] | :: of 'a * ('a list)
                (* if this were valid syntax *)

datatype 'a option = NONE | SOME of 'a
```

- `list`, `tree`, etc. are **not types**; they are ***type constructors***
- `int list`, `(string,real) tree`, etc. are types.
- Pattern-matching works on all datatypes.

The Value Restriction Appears ☹

If you use partial application to create a polymorphic function, it may not work due to the value restriction

- Warning about “type vars not generalized”
  - And won’t let you call the function
- This should surprise you; you did nothing wrong ☺ but you still must change your code
- See the written lecture summary about how to work around this wart (and ignore the issue until it arises)
- The wart is there for good reasons, related to mutation and not breaking the type system

Purpose of the Value Restriction

```
val xs = ref []
       (* xs : 'a list ref *)
val _  = xs := ["hi"]
       (* instantiate 'a with string *)
val y  = 1 + (hd (!xs))
       (* BAD: instantiate 'a with int *)
```

- A binding is only allowed to be polymorphic if the right-hand side is:
  - a variable; or
  - a value (including function definitions, constructors, etc.)
- `ref []` is not a value, so we can only give it non-polymorphic types such as `int list ref` or `string list ref`, but not `'a list ref`.

Downside of the Value Restriction

```
val pr_list = List.map (fn x => (x,x)) (* X *)

val pr_list : int list -> (int*int) list =
      List.map (fn x => (x,x))

val pr_list =
   fn lst => List.map (fn x => (x,x)) lst

fun pr_list lst = List.map (fn x => (x,x)) lst
```

- The SML type checker does not know if the `'a list` type uses references internally, so it has to be *conservative* and assume it could.
- In practice, this means we need to be more explicit about partial application of polymorphic functions.

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2                    t1 = t5  list
 xs : t1
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2                t1 = t5  list
 xs : t1                      t2 = int
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
```

```
t1 = t5  list
t2 = int
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2                    t1 = t5  list
 xs : t1                          t2 = int
  x : t3
xs' : t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5  list
t2 = int
t3 = t5
```

Type inference: sum

```
fun sum xs =
   case xs of
       [] => 0
     | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5 list
t2 = int
t3 = t5
t4 = t5 list
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5  list
t2 = int
t3 = t5
t4 = t5  list
t3 = int
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5  list
t2 = int
t3 = t5
t4 = t5  list
t3 = int
t1 = t4
```

Type inference: sum

```
fun sum xs =
  case xs of
       [] => 0
     | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t1
```

```
t1 = t5 list
t2 = int
t3 = t5
t4 = t5 list
t3 = int
t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t1
```

```
t1 = t5 list
t2 = int
t3 = t5
t1 = t5 list
t3 = int
t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t1
```

```
t1 = t5 list
t2 = int
t3 = t5
t1 = t5 list
t3 = int
t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
         [] => 0
       | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t1
```

```
t1 = t5 list
t2 = int
t3 = t5
t1 = t5 list
t3 = int
t1 = t4
```

Type inference: sum

```
fun sum xs =
  case xs of
      [] => 0
    | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : int
xs' : t1
```

```
t1 = t5 list
t2 = int
t3 = t5
t1 = t5 list
t3 = int
t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
         [] => 0
       | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : int
xs' : t1
```

```
 t1 = t5 list
 t2 = int
int = t5
 t1 = t5 list
 t3 = int
 t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : int
xs' : t1
```

```
 t1 = t5  list
 t2 = int
int = t5
 t1 = t5  list
 t3 = int
 t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2                t1 = int list
 xs : t1                      t2 = int
  x : int                    int = t5
xs' : t1                      t1 = t5  list
                              t3 = int
                              t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> t2
 xs : t1
  x : int
xs' : t1
```

```
 t1 = int list
 t2 = int
int = t5
 t1 = t5  list
 t3 = int
 t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> int
 xs : t1
  x : int
xs' : t1
```

```
 t1 = int list
 t2 = int
int = t5
 t1 = t5  list
 t3 = int
 t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum : t1 -> int
 xs : t1
  x : int
xs' : t1
```

```
 t1 = int list
 t2 = int
int = t5
 t1 = t5  list
 t3 = int
 t1 = t4
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum xs')
```

```
sum  : int list -> int          t1 = int list
 xs  : int list                 t2 = int
  x  : int                     int = t5
xs'  : int list                 t1 = t5  list
                                t3 = int
                                t1 = t4
```

Type inference: length

```
fun length xs =
   case xs of
       [] => 0
     | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2
    xs : t1
```

Type inference: length

```
fun length xs =
   case xs of
         [] => 0
       | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2                    t1 = t4 list
    xs : t1
```

Type inference: length

```
fun length xs =
   case xs of
         [] => 0
       | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2                    t1 = t4 list
    xs : t1                          t2 = int
```

Type inference: length

```
fun length xs =
   case xs of
        [] => 0
      | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2              t1 = t4 list
    xs : t1                    t2 = int
   xs' : t3
```

Type inference: length

```
fun length xs =
   case xs of
        [] => 0
      | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2                t1 = t4 list
    xs : t1                      t2 = int
   xs' : t3                      t3 = t4 list
```

Type inference: length

```
fun length xs =
   case xs of
        [] => 0
      | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2
    xs : t1
   xs' : t3
```

```
t1 = t4 list
t2 = int
t3 = t4 list
t1 = t3
```

Type inference: length

```
fun length xs =
   case xs of
      [] => 0
    | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2        t1 = t4 list
    xs : t1              t2 = int
   xs' : t1              t1 = t4 list
                         t1 = t3
```

Type inference: length

```
fun length xs =
   case xs of
         [] => 0
       | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2                    t1 = t4 list
    xs : t1                          t2 = int
   xs' : t1                          t1 = t4 list
                                     t1 = t3
```

Type inference: length

```
fun length xs =
   case xs of
         [] => 0
       | _::xs' => 1 + (length xs')
```

```
length : t1 -> t2                   t1 = t4 list
    xs : t1                         t2 = int
   xs' : t1                         t1 = t4 list
                                    t1 = t3
```

Type inference: length

```
fun length xs =
   case xs of
        [] => 0
      | _::xs' => 1 + (length xs')
```

```
length : t1 -> int
    xs : t1
   xs' : t1
```

```
t1 = t4 list
t2 = int
t1 = t4 list
t1 = t3
```

Type inference: length

```
fun length xs =
   case xs of
        [] => 0
      | _::xs' => 1 + (length xs')
```

```
length : t1 -> int
    xs : t1
   xs' : t1
```

```
t1 = t4 list
t2 = int
t1 = t4 list
t1 = t3
```

Type inference: length

```
fun length xs =
   case xs of
        [] => 0
      | _::xs' => 1 + (length xs')
```

```
length : t4 list -> int        t1 = t4 list
    xs : t4 list -> int        t2 = int
   xs' : t4 list               t1 = t4 list
                               t1 = t3
```

Type inference: length

```
fun length xs =
   case xs of
        [] => 0
      | _::xs' => 1 + (length xs')
```

```
length : 'a list -> int          t1 = t4 list
    xs : t4 list -> int          t2 = int
   xs' : t4 list                 t1 = t4 list
                                 t1 = t3
```

`length` works no matter what `'a` is.

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2                    t3 = t4 -> t5
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2                    t3 = t4 -> t5
      x : t4
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2                    t3 = t4 -> t5
      x : t4                    t2 = t4 -> t6
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2                         t3 = t4 -> t5
      x : t4                         t2 = t4 -> t6
                                     t1 = t6 -> t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2                  t3 = t4 -> t5
      x : t4                  t2 = t4 -> t6
                              t1 = t6 -> t7
                              t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2                    t3 = t4 -> t5
      x : t4                    t2 = t4 -> t6
                                t1 = t6 -> t5
                                t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : t1 * t2 -> t3
      f : t1
      g : t2                         t3 = t4 -> t5
      x : t4                         t2 = t4 -> t6
                                     t1 = t6 -> t5
                                     t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : (t6 -> t5) * t2 -> t3
      f : t6 -> t5
      g : t2                    t3 = t4 -> t5
      x : t4                    t2 = t4 -> t6
                                t1 = t6 -> t5
                                t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : (t6 -> t5) * t2 -> t3
      f : t6 -> t5
      g : t2                        t3 = t4 -> t5
      x : t4                        t2 = t4 -> t6
                                    t1 = t6 -> t5
                                    t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : (t6 -> t5) * (t4 -> t6) -> t3
      f : t6 -> t5
      g : t4 -> t6                  t3 = t4 -> t5
      x : t4                        t2 = t4 -> t6
                                    t1 = t6 -> t5
                                    t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : (t6 -> t5) * (t4 -> t6) -> t3
      f : t6 -> t5
      g : t4 -> t6
      x : t4
```

```
t3 = t4 -> t5
t2 = t4 -> t6
t1 = t6 -> t5
t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : (t6 -> t5) * (t4 -> t6) -> (t4 -> t5)
      f : t6 -> t5
      g : t4 -> t6                t3 = t4 -> t5
      x : t4 -> t5                t2 = t4 -> t6
                                  t1 = t6 -> t5
                                  t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : (t6 -> t5) * (t4 -> t6) -> (t4 -> t5)
      f : t6 -> t5
      g : t4 -> t6                 t3 = t4 -> t5
      x : t4 -> t5                 t2 = t4 -> t6
                                   t1 = t6 -> t5
                                   t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : ('a -> 'b) * ('c -> 'a) -> ('c -> 'b)
        f : t6 -> t5
        g : t4 -> t6                    t3 = t4 -> t5
        x : t4 -> t5                    t2 = t4 -> t6
                                        t1 = t6 -> t5
                                        t5 = t7
```

Type inference: compose

```
fun compose (f,g) = fn x => f (g x)
```

```
compose : ('a -> 'b) * ('c -> 'a) -> ('c -> 'b)
      f : t6 -> t5
      g : t4 -> t6                  t3 = t4 -> t5
      x : t4 -> t5                  t2 = t4 -> t6
                                    t1 = t6 -> t5
                                    t5 = t7

compose : ('b -> 'c) * ('a -> 'b) -> ('a -> 'c)
```

Type inference: broken sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
```

Type inference: broken sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2                    t1 = t5  list
 xs : t1
```

Type inference: broken sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2                  t1 = t5  list
 xs : t1                        t2 = int
```

Type inference: broken sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t3
```

```
t1 = t5  list
t2 = int
```

Type inference: broken sum

```
fun sum xs =
  case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5 list
t2 = int
```

Type inference: broken sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5  list
t2 = int
t3 = t5
```

Type inference: broken sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5 list
t2 = int
t3 = t5
t4 = t5 list
```

Type inference: broken sum

```
fun sum xs =
  case xs of
       [] => 0
     | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5  list
t2 = int
t3 = t5
t4 = t5  list
t3 = int
```

Type inference: broken sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t3
xs' : t4
```

```
t1 = t5  list
t2 = int
t3 = t5
t4 = t5  list
t3 = int
t1 = t3
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t1
xs' : t4
```

```
t1 = t5 list
t2 = int
t1 = t5
t4 = t5 list
t1 = int
t1 = t3
```

Type inference: sum

```
fun sum xs =
   case xs of
       [] => 0
     | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
 xs : t1
  x : t1
xs' : t4
```

```
t1 = t5 list
t2 = int
t1 = t5
t4 = t5 list
t1 = int
t1 = t3
```

Type inference: sum

```
fun sum xs =
   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : int-> t2
 xs : int
  x : int
xs' : t4
```

```
int = t5  list
 t2 = int
int = t5
 t4 = t5  list
 t1 = int
 t1 = t3
```

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Type inference: sum

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   case xs of
       [] => 0
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```

```
sum : int-> t2          int = t5  list
 xs : int                t2 = int
  x : int               int = t5
xs' : t5  list           t4 = t5  list
                         t1 = int
                         t1 = t3
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 t1 = t3
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```
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   case xs of
        [] => 0
      | x::xs' => x + (sum x)
```

```
sum : int-> t2
 xs : int
  x : int
xs' : int list
```

```
$\boxed{\text{int = int list}}$
 t2 = int
int = t5
 t4 = t5  list
 t1 = int
 t1 = t3
```

Type inference: sum

```
fun sum xs =
   case xs of
         [] => 0
       | x::xs' => x + (sum x)
```

```
sum  : int-> t2
 xs  : int
  x  : int
xs'  : int list
```

$\boxed{\texttt{int = int list}}$

```
 t2 = int
int = t5
 t4 = t5  list
 t1 = int
 t1 = t3
```

Type inference: sum

```
fun sum xs =
   case xs of
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      | x::xs' => x + (sum x)
```

```
sum  : int-> int
 xs  : int
  x  : int
xs'  : int list
```

```
int = int list
 t2 = int
int = t5
 t4 = t5  list
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Type inference: sum

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sum  : int-> int
 xs  : int
  x  : int
xs'  : int list
```

```
int = int list
 t2 = int
int = t5
 t4 = t5  list
 t1 = int
 t1 = t3
```

Parting comments on ML type inference

- You almost never have to write types in ML (even on parameters), with some minor caveats.
- Hindley-Milner type inference algorithm
- ML has no subtyping. If it did, the equality constraints we used for inference would be overly restrictive.
- Type variables and inference are not tied to each. Some languages have one without the other.
  - Type variables alone allow convenient code reuse.
  - Without type variables, we cannot give a type to compose until we see it used.