

CSE 341: Programming Languages

Autumn 2005

Lecture 10 — Mutual Recursion, Equivalence, and Syntactic Sugar

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Mutual Recursion Idioms

1. Encode a state machine (see `product_sign` example)
 - Sometimes easier to understand than explicit state values.
2. Process mutually recursive types, as in HW 2

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Mutual Recursion

You've already seen how multiple functions can recursively call each other in HW 2.

ML uses the keyword `and` to provide different *scope* rules. Example:

```
fun even i = if i=0 then true  else odd  (i-1)
and odd  i = if i=0 then false else even (i-1)
```

Roughly extends the binding form for functions from `fun f1 x1 = e1 to fun f1 x1 = e1 and f2 x2 = e2 and ... and fn xn = en.`

Syntax gotcha: Easy to forget that you write `and fi xi = ei`, not `and fun fi xi = ei`.

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Equivalence

"Equivalence" is a fundamental programming concept

- Code maintenance / backward-compatibility
- Program verification
- Program optimization
- Abstraction and strong interfaces

But what does it mean for an expression (or program) `e1` to be "equivalent" to expression `e2`?

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Equivalence I: where?

Context (i.e., “where equivalent”)

- Given where $e1$ occurs in a program e , replacing $e1$ with $e2$ makes a program e' equivalent to e
- At any point in any program, replacing $e1$ with $e2$ makes an equivalent program.

The latter (contextual equivalence) is much more interesting.

For the former, the body of an unused function body is equivalent to everything (that typechecks).

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Accounting for “Effects”

Consider whether $\text{fn } x \Rightarrow e1$ and $\text{fn } x \Rightarrow e2$ are totally contextually equivalent.

Is this enough? For all environments, $e1$ terminates and evaluates to v under the environments if and only if $e2$ terminates and evaluates to v under the environment.

Functions produce *values*; may also produce (*side-*) *effects*. Consider both!

Purely functional languages have fewer/none, but ML is not purely functional.

In real languages, contextual equivalence usually requires many things.

Nonetheless, “equivalence” usually means total contextual equivalence for practical purposes (optimization, reasoning about correctness, etc.).

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Equivalence II: how?

“how equivalent”

- “partial”: e and e' are equivalent if they input and output the same data (any limits on input?)
- “total”: partial plus e and e' have the same termination behavior
- efficiency: e and e' are totally equivalent and one never takes more than (for example) c times longer than the other (or uses much more space or ...)
- syntactic notions: e and e' differ only in whitespace and comments (for example)

Key notion: what is observable? (memory, clock, REP-loop, file-system, ...)

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Syntactic Sugar

“Syntactic sugar causes cancer of the semicolon.”

– Alan Perlis

When all expressions using one construct are totally equivalent to another more primitive construct, we say the former is “syntactic sugar”.

- Makes language definition easier
- Makes language implementation easier

Examples:

- `e1 andAlso e2` (define as a conditional)
- `if e1 then e2 else e3` (define as a case)
- `fun f x y = e` (define with an anonymous function)

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More sugar

#1 `e` is just `let val (x,...) = e in x end`

If we *ignore types*, then we have even more sugar:

`let val p = e1 in e2 end` is just `(fn p => e2) e1`.

In fact, if we let every program type-check (or just use one big datatype), then a language with just functions and function application is as powerful as ML or Java (in the Turing Tarpit sense).

This language is called “lambda calculus” – we’ll learn a bit more about it later.

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Equivalences for Functions

While sugar defines one construct in terms of another, there are also important notions of *meaning-preserving* changes involving functions and bound variables.

They’re so important that a goal of language design is that a language supports them.

But the correct definitions are subtle.

First example: systematic renaming

Is `fn x => e1` equivalent to `fn y => e2` where `e2` is `e1` with every `x` replaced by `y`?

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Systematic renaming requires care

Is `fn x => e1` equivalent to `fn y => e2` where `e2` is `e1` with every `x` replaced by `y`?

What if `e1` is `y`?

What if `e1` is `fn x => x`?

Need caveats: `fn x => e1` is equivalent to `fn y => e2` where `e2` is `e1` with every *free* `x` replaced by `y` and `y` is not *free* in `e1`, and no free `x` occurs within the scope of a binding for `y` (capture; e.g.: `fn x => let y = 2 in x + y end`; see also next slide.)

Note: We can provide a very precise recursive (meta-)definition of *free variables* in an expression.

Next: Is `(fn x => e1) e2` equivalent to `e3` where `e3` is `e1` with every `x` replaced by `e2`?

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Argument Substitution

Is `(fn x => e1) e2` equivalent to `e3` where `e3` is `e1` with every `x` replaced by `e2`?

- Every *free* `x` (of course).
- A free variable in `e2` must not be bound at an occurrence of `x`. (Called “capture”.)
 - Always satisfiable by renaming bound variables.
- Evaluating `e2` must have no effects (printing, exceptions, infinite-loop, etc.)
 - Closely tied to the rule that arguments are evaluated to values *before* function application. (Not true for all languages)
 - In ML, many expressions have no such effects (`x, #foo x, ...`); much fewer in Java.
- Efficiency? Could be faster or slower. (Why?)

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Unnecessary Function Wrapping

A common source of bad style for beginners

Is `e1` equivalent to `fn x => e1 x`?

Sure, provided:

- `e1` is effect-free
- `x` does not occur free in `e1`

Example:

```
List.map (fn x => SOME x) lst
```

```
List.map SOME lst
```

Summary

We breezed through some core programming-language facts and design goals:

- Definition of equivalence depends on observable behavior
- Syntactic sugar “makes a big language smaller” by *defining* constructs in terms of equivalence
- Notion of free and bound variables crucial to understanding function equivalence.
- Three common forms of function equivalence:
 - Systematic Renaming
 - Argument Substitution
 - Unnecessary Function Wrapping