CSE 332 Winter 2024
Lecture 20: Parallel Prefix

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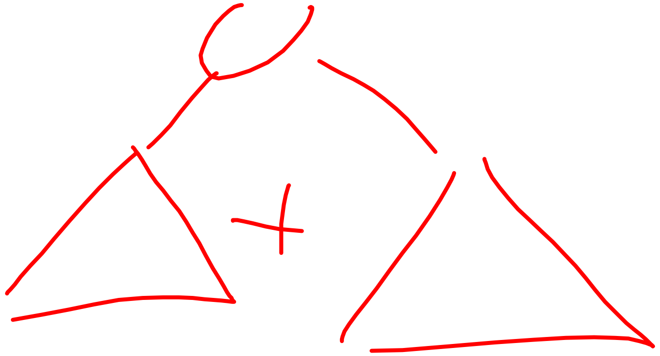
Which Data Structures are "Suitable" for Parallelism?

- For each data structure, can we write a parallel algorithm to sum all of its values that's *more efficient* than a sequential one?
  - Array
  - Linked List
  - Tree

indexing

No



ForkJoin Framework

- This strategy is common enough that Java (and C++, and C#, and…) provides a library to do it for you!

What you would do in Threads	What to instead in ForkJoin
Subclass **Thread**	Subclass **RecursiveTask<V>**
Override **run**	Override **compute**
Store the answer in a field	Return a V from compute
Call **start**	Call **fork**
join synchronizes only	**join** synchronizes and returns the answer
Call **run** to execute sequentially	Call **compute** to execute sequentially
Have a topmost thread and call **run**	Create a pool and call **invoke**

Divide and Conquer with ForkJoin

```
class SumTask extends RecursiveTask<Integer> {
        int lo; int hi; int[] arr; // fields to know what to do
        SumTask(int[] a, int l, int h) { … }
        protected Integer compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        int ans = 0; // local var, not a field
                        for(int i=lo; i < hi; i++) {
                                ans += arr[i]; return ans; }
                else {
                        SumTask left = new SumTask(arr,lo,(hi+lo)/2); // divide
                        SumTask right= new SumTask(arr,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        int rightAns = right.compute(); //call compute directly (conquer)
                        int leftAns = left.join(); // get result from left
                        return leftAns + rightAns; // combine
                }
        }
}
```

Divide and Conquer with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();
int sum(int[] arr){
	SumTask task = new SumTask(arr,0,arr.length)
	return POOL.invoke(task); // invoke returns the value compute returns
}
```

Find Max with ForkJoin

```
class MaxTask extends RecursiveTask<Integer> {
        int lo; int hi; int[] arr; // fields to know what to do
        SumTask(int[] a, int l, int h) { … }
        protected Integer compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        int ans = Integer.MIN_VALUE; // local var, not a field
                        for(int i=lo; i < hi; i++) {
                                ans = Math.max(ans, arr[i]);}
                        return ans;
                else {
                        MaxTask left = new MaxTask(arr,lo,(hi+lo)/2); // divide
                        MaxTask right= new MaxTask(arr,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        int rightAns = right.compute(); //call compute directly (conquer)
                        int leftAns = left.join(); // get result from left
                        return Math.max(rightAns, leftAns); // combine
                }
        }
}
```

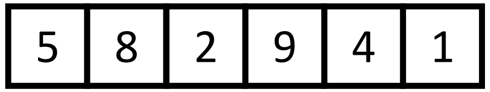
Other Problems that can be solved similarly

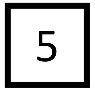
- Element Search
  - Is the value 17 in the array?
- Counting items with a certain property
  - How many elements of the array are divisible by 5?
- Checking if the array is sorted
- Find the smallest rectangle that covers all points in the array
- Find the first thing that satisfies a property
  - What is the leftmost item that is divisible by 20?

Reduction/Fold

- All examples of a category of computation called a reduction (or fold)
  - We "reduce" all elements in an array to a single item
  - Requires operation done among elements is associative
    - $(x+y)+z = x+(y+z)$
  - The "single item" can itself be complex
    - E.g. create a histogram of results from an array of trials

Reduction (sum an array)



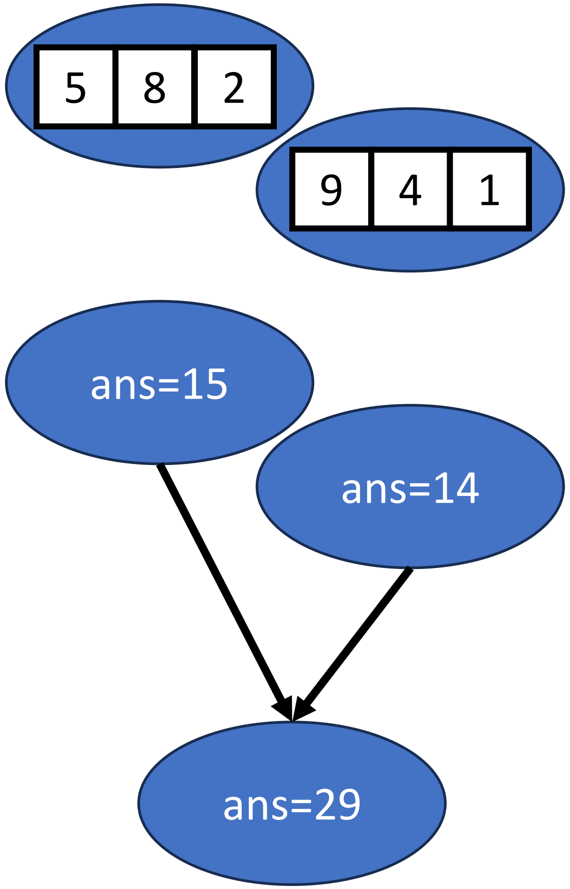


- **Base Case:**
  - If the list's length is smaller than the Sequential Cutoff, reduce things sequentially

- **Divide:**
  - Split the list into two "sublists" of (roughly) equal length, create a thread to reduce each sublist.

- **Conquer:**
  - Call **start()** for each thread

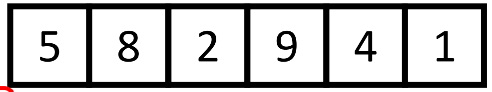
- **Combine:**
  - Reduce the answers from each thread

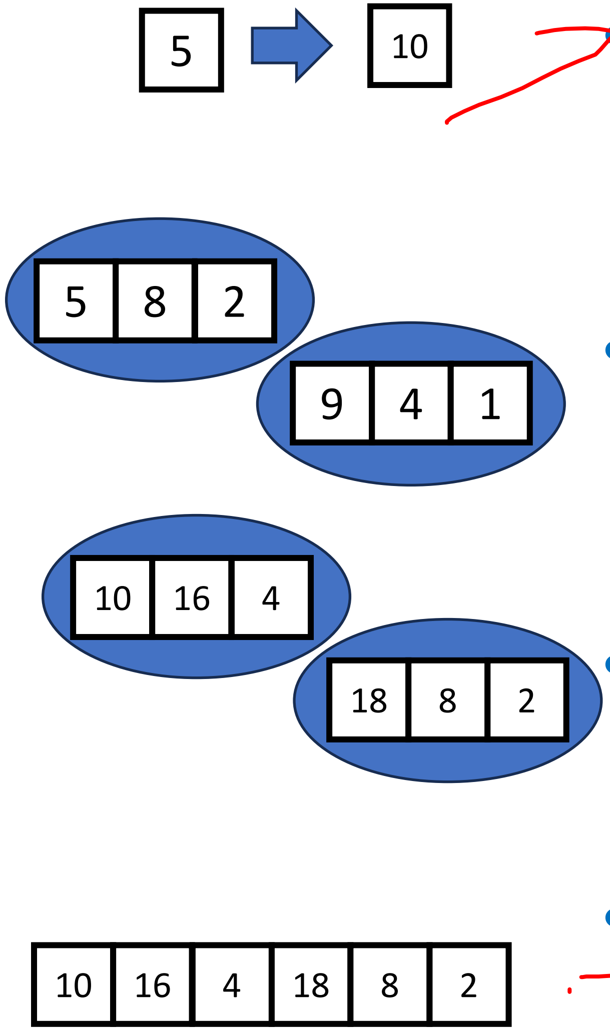


Map

- Perform an operation on each item in an array to create a new array of the same size
- Examples:
  - Vector addition:
    - sum[i] = arr1[i] + arr2[i]
  - Function application:
    - out[i] = f(arr[i]);

Map (double each value)





- **Base Case:**
  - If the list's length is smaller than the Sequential Cutoff, convert each thing sequentially
- **Divide:**
  - Split the list into two "sublists" of (roughly) equal length, create a thread to map each sublist.
- **Conquer:**
  - Call **start()** for each thread
- **Combine:**
  - No additional work necessary

Map with ForkJoin

```
class AddTask extends RecursiveAction {
        int lo; int hi; int[] arr; // fields to know what to do
        AddTask(int[] a, int[] b, int[] sum, int l, int h) { … }
        protected void compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        for(int i=lo; i < hi; i++) {
                                sum[i] = a[i] + b[i];}
                else {
                        AddTask left = new AddTask(a,b,sum,lo,(hi+lo)/2); // divide
                        AddTask right= new AddTask(a,b,sum,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        right.compute(); //call compute directly (conquer)
                        left.join(); // get result from left
                        return; // combine
                }
        }
}
```

Map with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();
Int[] add(int[] a, int[] b){
        ans = new int[a.length];
        AddTask task = new AddTask(a, b, ans, 0, a.length)
        POOL.invoke(task);
        return ans;
}
```

Maps and Reductions

- “Workhorse” constructs in parallel programming
- Many problems can be written in terms of maps and reductions
- With practice, writing them will become second nature
  - Like how over time for loops and if statements have gotten easier

Map/Reduction Example

- Multiply together the lengths of all of the odd-length strings in a given array
  - First, do a map to covert the array of strings into an array of their lengths
  - Then do a map on that array so each value maps to 1 if it's even and itself if it's odd
  - Then do a reduction to multiply together that final result
- Note: You could do this in a single ForkJoin RecursiveTask, but it's worthwhile to recognize how to "deconstruct" it since some programming languages designed specifically for parallelism have Map/Reduce built in.
  - Map and Reduce are two from a trio, with Pack/Filter being the third

Pack/Filter

- Given an array of values and a Boolean function, return a new array which contains only elements that were “true”

10	16	4	18	8	2

$\Rightarrow$

10	16	18

$f(x) = x > 9$

Prefix Sum

- Given an array, compute a new array where each index i is the sum of all values up to i

10	16	4	18	8	2

10	26	30	48	56	58

i

```
int[] prefixSum(int[] arr){
        int[] output = new int[arr.length];
        output[0] = arr[0];
        for (int i = 1; I < arr.length, i++)
                output[i] = output[i-1] + arr[i];
        return output;
}
```

Parallel Prefix Sum

- Algorithm will have two major parallel steps
  - Called a “two pass” parallel algorithm
- First step:
  - Create a tree data structure
- Second Step:
  - Use the tree to fill in the output array

Richard Ladner
Allen School Faculty

Tree Node:

range: [lo,hi)
sum:
leftSum:

The “subproblem” this node represents
lower bound is inclusive, upper is exclusive

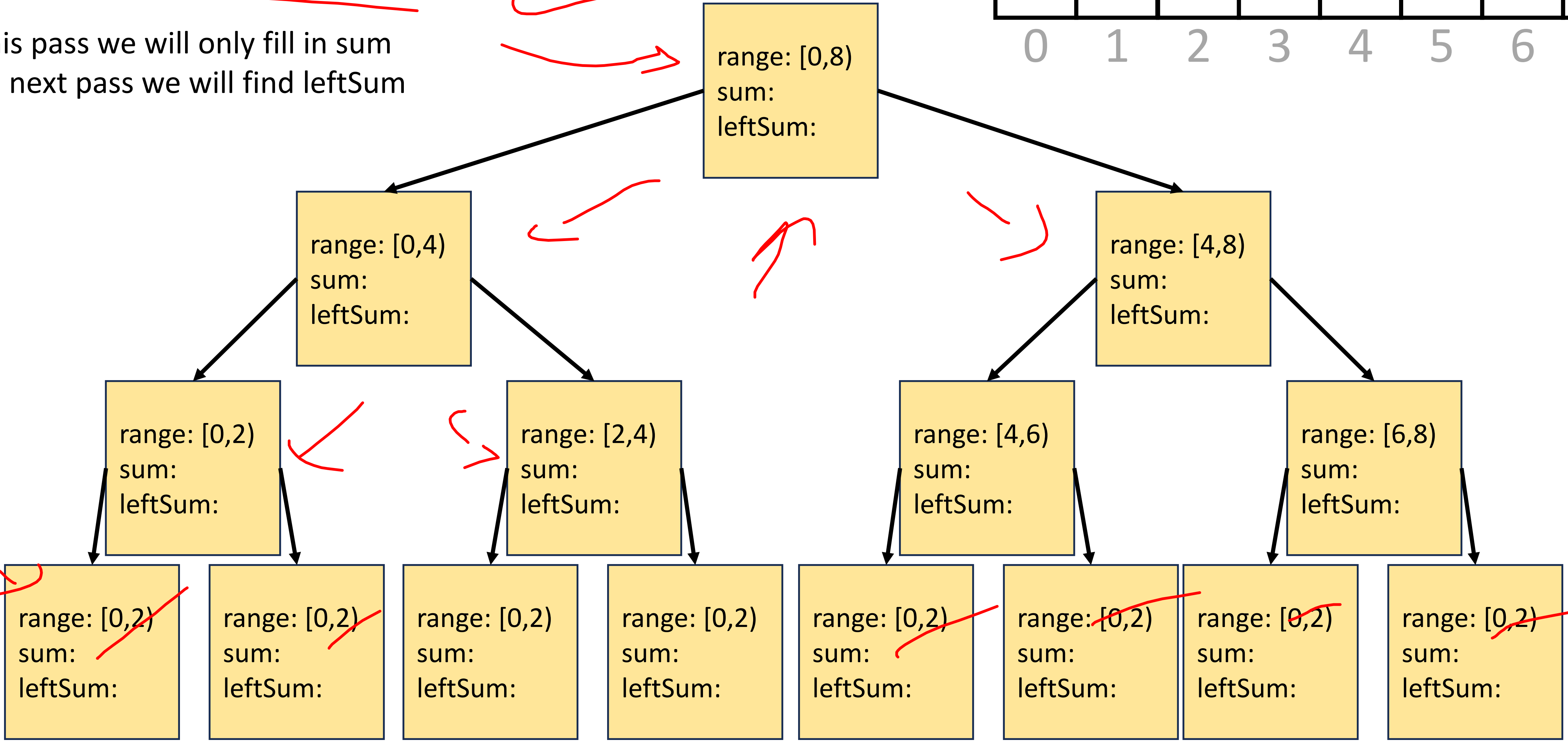
The sum of all values in the range

The sum of all values to the left of the range
i.e. in the range [0, lo)

Step 1: Using D&C
Create a Tree, Fill in sum

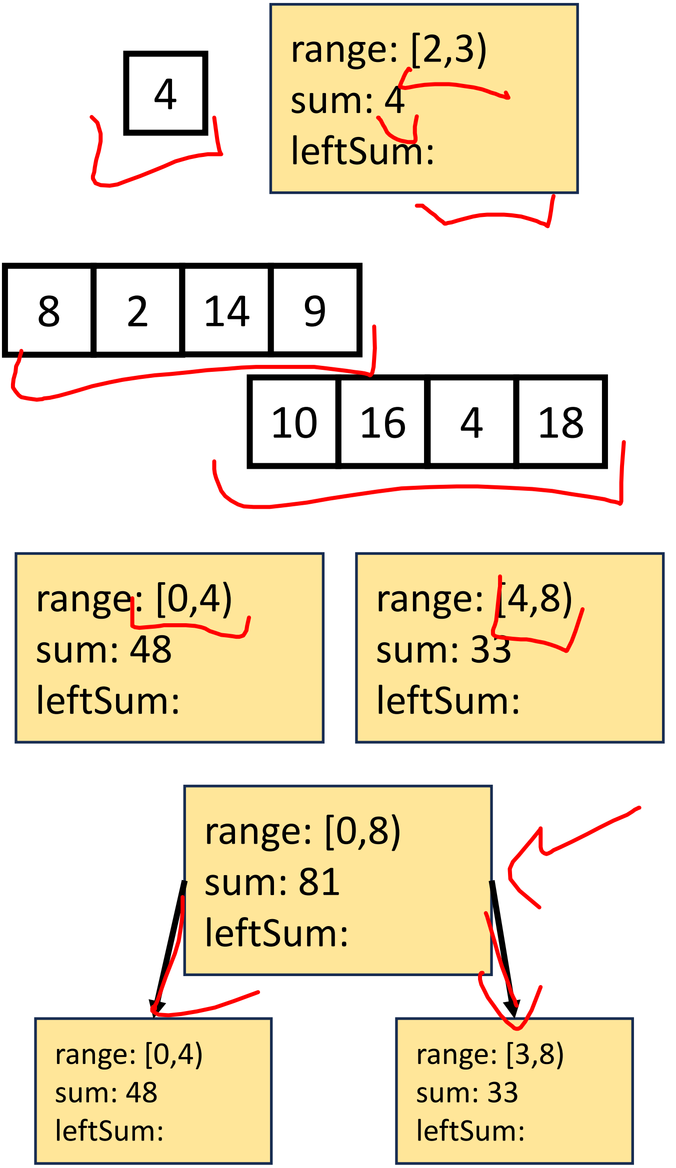
For this pass we will only fill in sum
In the next pass we will find leftSum

	0	1	2	3	4	5	6	7
Input:	10	16	4	18	8	2	14	9
Output:								



Step 1: Create a Tree, Fill in sum

10	16	4	18	8	2	14	9



- **Base Case**:
  - If the rand is smaller than the Sequential Cutoff, create a node for that range and find the sum sequentially
- **Divide**:
  - Split the list into two “sublists” of (roughly) equal length, create a thread for each sublist.
- **Conquer**:
  - Call **start()** for each thread to compute the left and right subtrees
- **Combine**:
  - Create parent node, connect to children, fill in sum

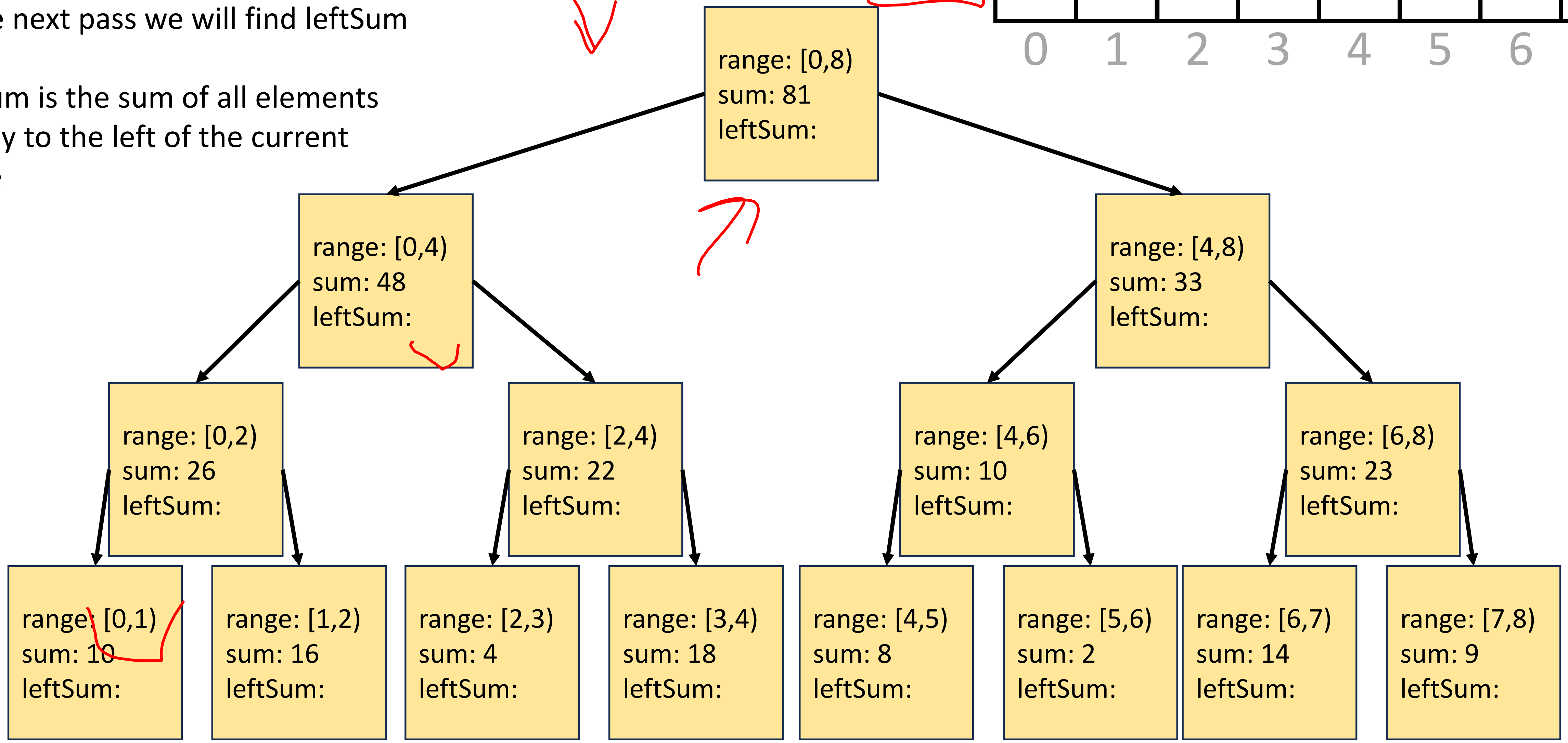
```
class BuildTree extends RecursiveTask<PrefixSumNode> {
        protected PrefixSumNode compute(){
                if(hi – lo < SEQUENTIAL_CUTOFF) { // base case
                        int ans = 0; // local var, not a field
                        for(int i=lo; i < hi; i++)
                                ans += arr[i];
                        return new PrefixSumNode(lo, hi, ans); }
                else {
                        BuildTree left = new BuildTree(arr,lo,(hi+lo)/2);
                        BuildTree right= new BuildTree(arr,(hi+lo)/2,hi);
                        left.fork();
                        PrefixSumNode rightChild = right.compute();
                        PrefixSumNode leftChild = left.join();
                        int ans = rightChild.sum + leftChild.sum;
                        parent = new PrefixSumNode(lo, hi, ans);
                        parent.left = leftChild;
                        parent.right = rightChild;
                        return parent; }
        }
}
```

After Step 1

All sums filled in per node
In the next pass we will find leftSum

leftSum is the sum of all elements strictly to the left of the current range

	0	1	2	3	4	5	6	7
Input:	10	16	4	18	8	2	14	9
Output:								

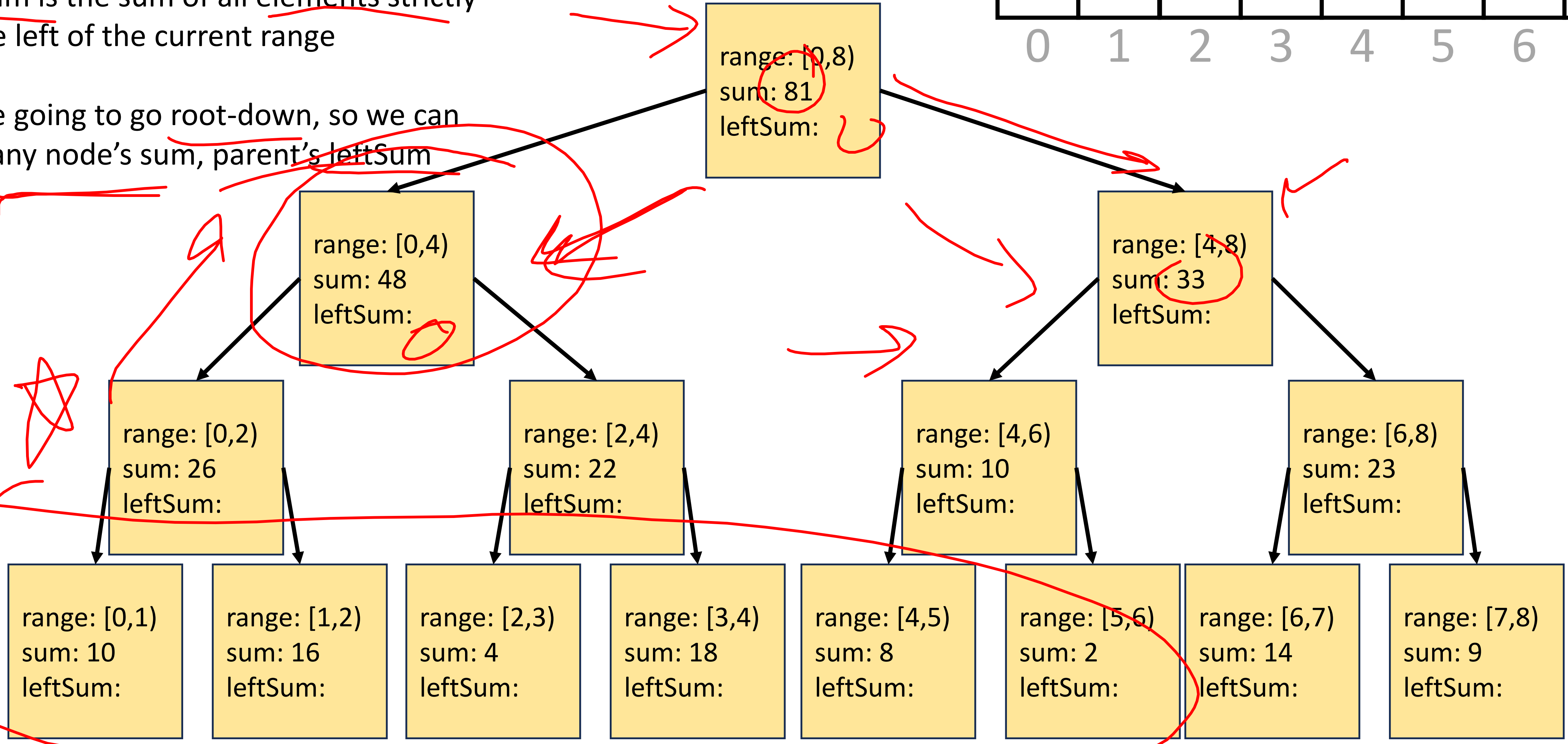


Step 2: fill in leftSum and Output

leftSum is the sum of all elements strictly to the left of the current range

We're going to go root-down, so we can use: any node's sum, parent's leftSum

Input:	10	16	4	18	8	2	14	9
Output:								
	0	1	2	3	4	5	6	7



Step 2: fill in leftSum and Output

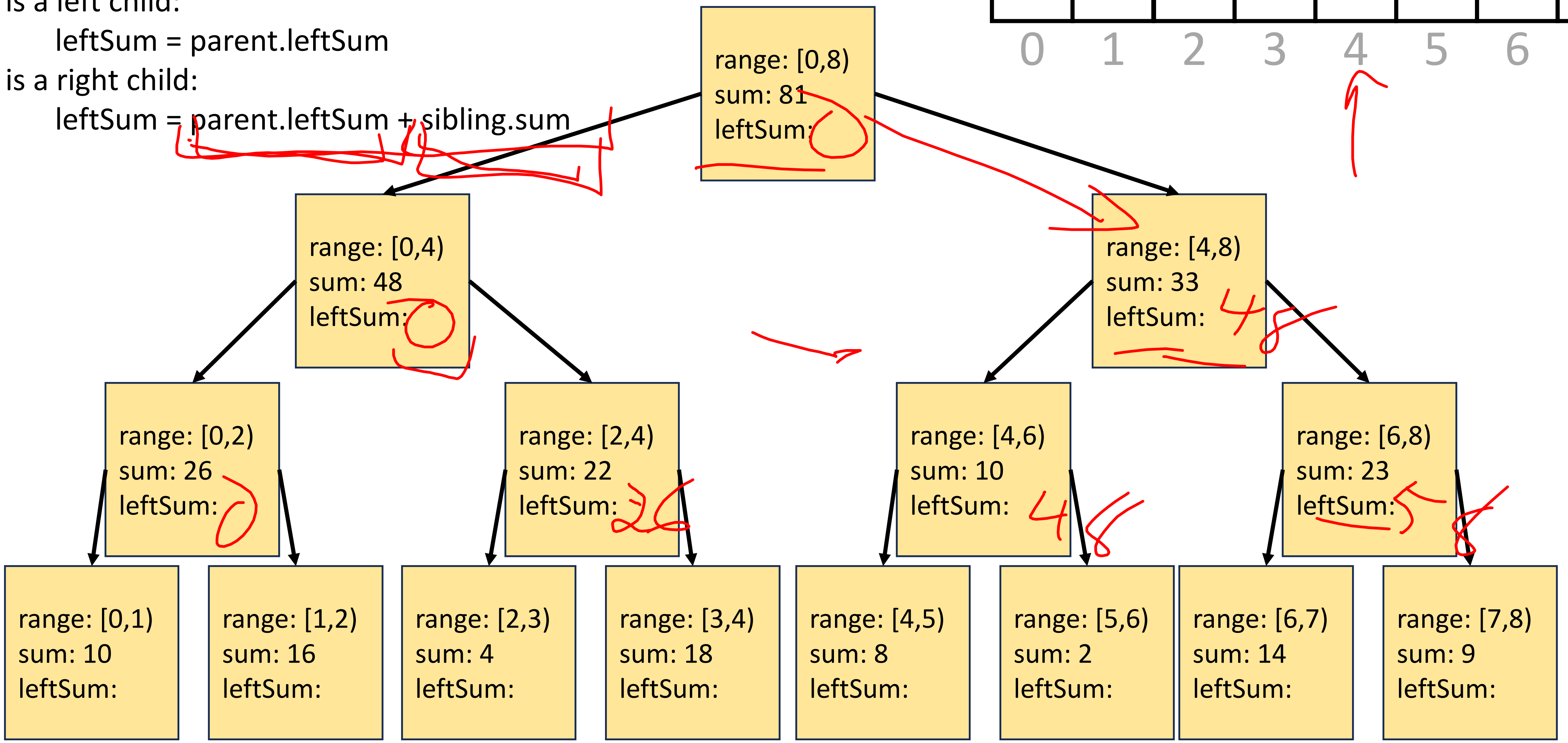
If this is a left child:

    leftSum = parent.leftSum

If this is a right child:

    leftSum = parent.leftSum + sibling.sum

	0	1	2	3	4	5	6	7
Input:	10	16	4	18	8	2	14	9
Output:								



Step 2: fill in leftSum and Output

Input:	10	16	4	18	8	2	14	9
Output:	10	26	30					
	0	1	2	3	4	5	6	7

If this is a left child:

leftSum = parent.leftSum

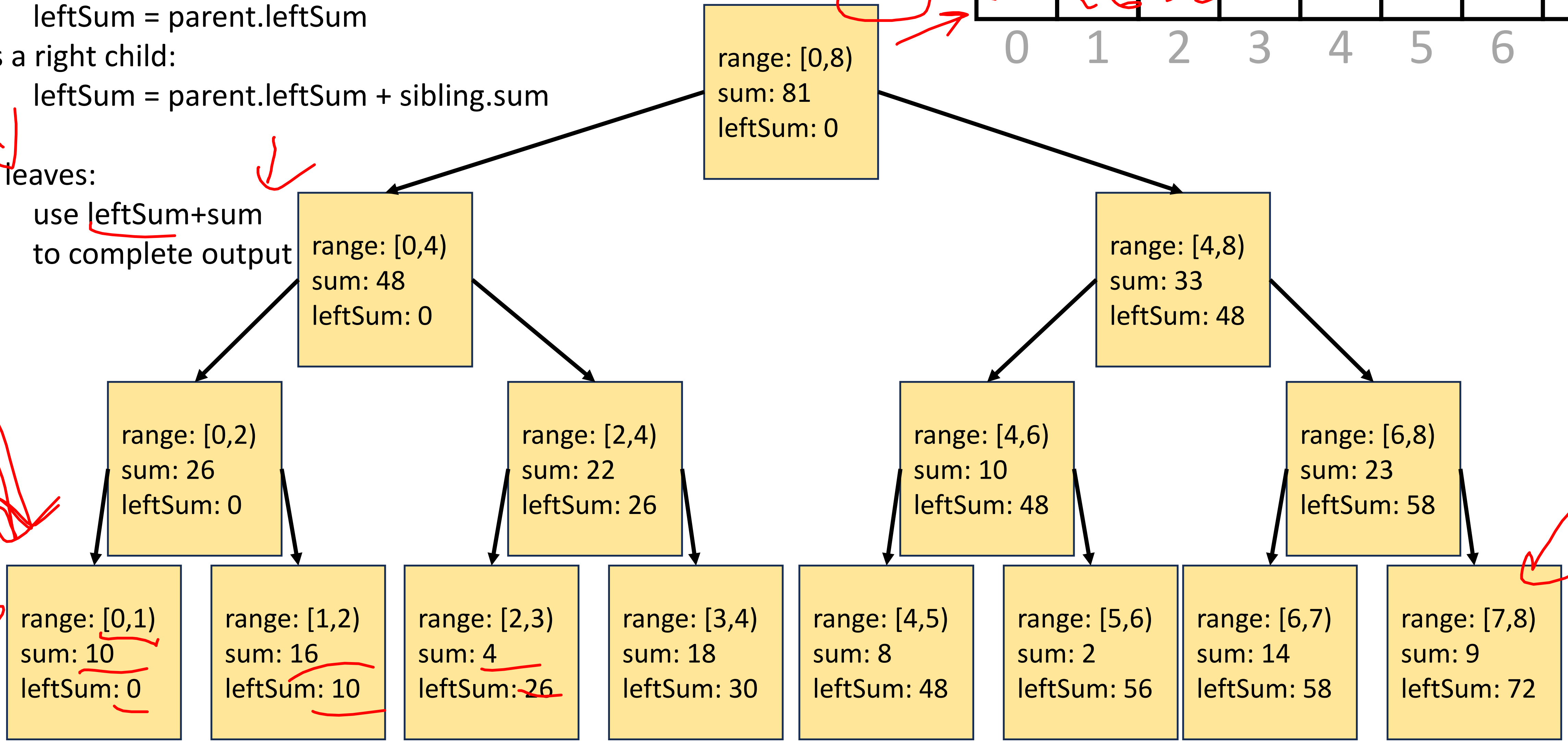
If this is a right child:

leftSum = parent.leftSum + sibling.sum

For the leaves:

use leftSum+sum

to complete output



Step 2: fill in leftSum and Output

Input:	10	16	4	18	8	2	14	9
Output:	10	26	30	48	56	58	72	81
	0	1	2	3	4	5	6	7

If this is a left child:

leftSum = parent.leftSum

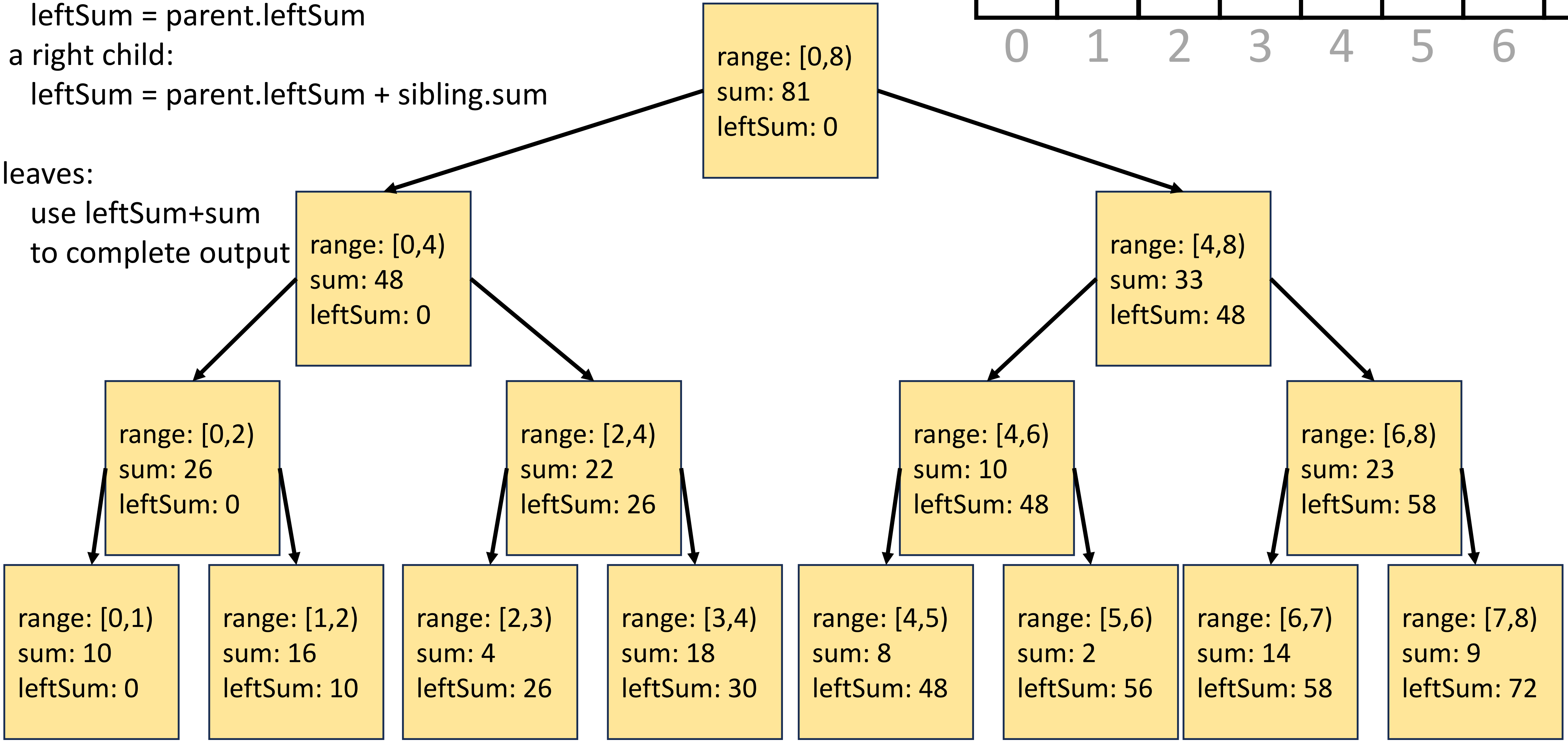
If this is a right child:

leftSum = parent.leftSum + sibling.sum

For the leaves:

use leftSum+sum

to complete output



```
class CompleteTree extends RecursiveAction {
        public CompleteTree(PrefixSumNode curr, PrefixSumNode parent, PrefixSumNode sibling, boolean isLeftChild, int[] output, int[] input){…}
        protected Void compute(){
                if(isLeftChild)
                        curr.sumLeft = parent.sumLeft;
                else
                        curr.sumLeft = parent.sumLeft + sibling.sum;
                if (curr.leftChild != Null && curr.rightChild != Null){  // if this isn't a leaf
                        CompleteTree left = new CompleteTree(curr.leftChild, curr, curr.rightChild, true, output, input);
                        left.fork();
                        CompleteTree right = new CompleteTree(curr.rightChild, curr, curr.leftChild, false, output, input);
                        right.compute();
                        left.join();
                }
                else{
                        output[curr.lo] = curr.sumLeft + input[curr.lo];
                        for(int i = curr.lo; i < curr.hi; i++){
                                output[i] = output[i-1] + input[i]
                        }
                }
        }
}
```

Whew! Back to Pack/Filter

- Given an array of values and a Boolean function, return a new array which contains only elements that were “true

10	16	4	18	8	2	14	9

$\Rightarrow$

10	16	18	14

$$f(x) = x > 9$$

Parallel Pack

Input:

10	16	4	18	8	2	14	9

, $f(x) = x > 9$

Output:

10	16	18	14

0 1 2 3 4 5 6 7

1. Do a map to identify the true elements

1	1	0	1	0	0	1	0

2. Do prefix sum on the result of the map to identify the count of true elements seen to the left of each position

1	2	2	3	3	3	4	4

3. Do a map using the previous results fill in the output

10	16	18	14

3. Do a map using the result of the prefix sum to fill in the output

Input:	10	16	4	18	8	2	14	9
Map Result:	1	1	0	1	0	0	1	0
Prefix Result:	1	2	2	3	3	3	4	4
Output:								

- Because the last value in the prefix result is 4, the length of the output is 4
- Each time there is a 1 in the map result, we want to include that element in the output
- If element i should be included, its position matches prefixResult[i]-1

```
Int[] output = new int[prefixResult[input.length-1]];
FORALL(int i = 0; i < input.length; i++){
        if (mapResult[i] == 1)
                output[prefixResult[i]-1] = input[i];
}
```

Map/Reduction/Pack Example

- Multiply together the lengths of all of the odd-length strings in a given array
  - First, do a map to covert the array of strings into an array of their lengths
  - Then do a map on that array so each value maps to 1 if it’s even and itself if it’s odd
    - Alternatively, do a pack on the array to remove all even values
  - Then do a reduction to multiply together that final result