CSE 332 Winter 2024
Lecture 13: Hashing and Sorting

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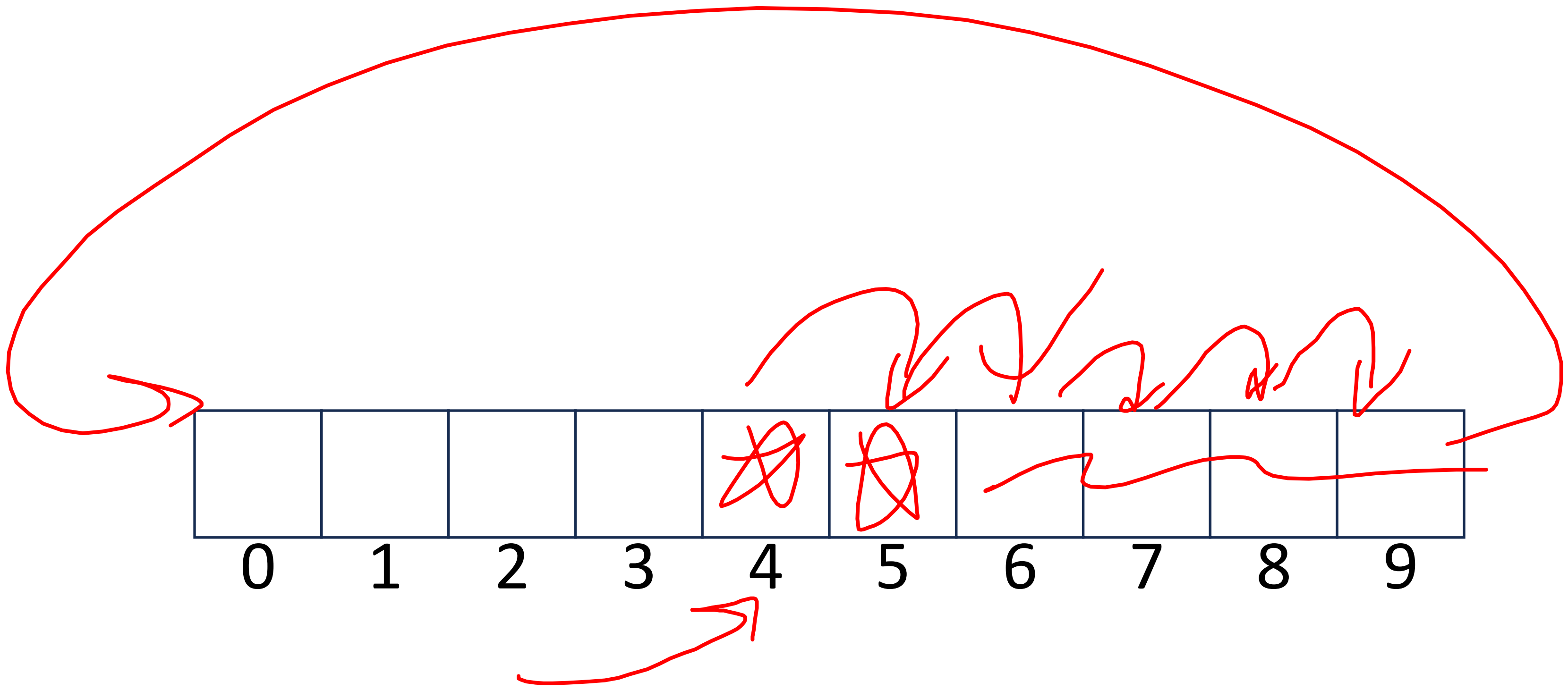
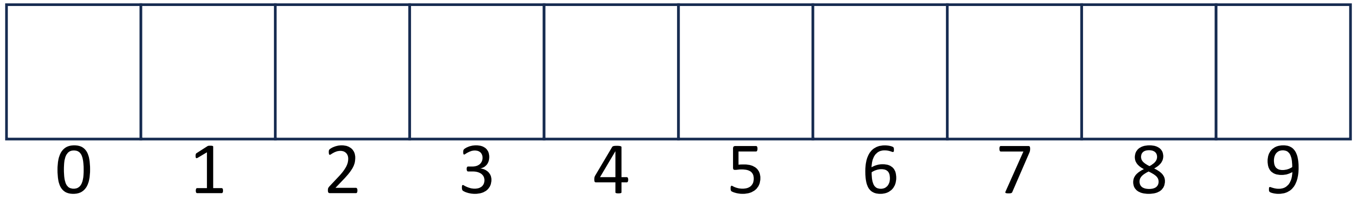
Collision Resolution: Linear Probing

- When there’s a collision, use the next open space in the table



Linear Probing: Insert Procedure

- To insert k, v
  - Calculate $i = h(k)\ \%arrsize$
  - If $table[i]$ is occupied then try $(i + 1)\%arrsize$
  - If that is occupied try $(i + 2)\%arrsize$
  - If that is occupied try $(i + 3)\%arrsize$
  - …



Linear Probing: Find

- $i = h(k)\%arrsize$
  - If i has the key or it's empty, then we're done
  - Otherwise:
    - Check $(i+1)\%arrsize$ if it's there, done else
    - Check $(i+2)\%arrsize$ if it's there, done else
    - Check $(i+3)\%arrsize$
    - …
    - Until we hit an empty cell

Linear Probing: Find

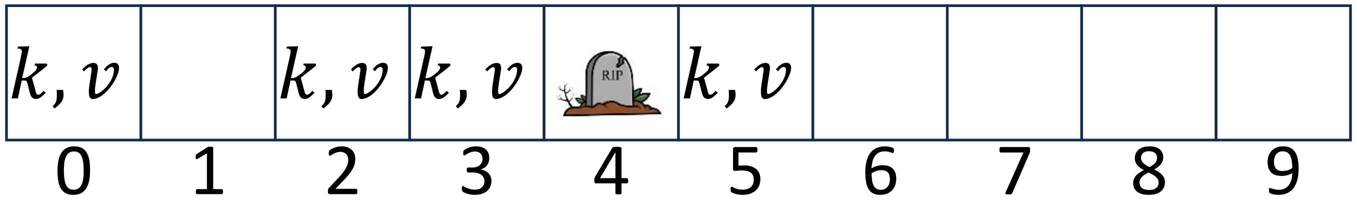
- To find key k
  - Calculate $i = h(k)\ \%arrsize$
  - If $table[i]$ is occupied and does not contain k then look at $(i+1)\ \%arrsize$
  - If that is occupied and does not contain k then look at $(i+2)\ \%arrsize$
  - If that is occupied and does not contain k then look at $(i+3)\ \%arrsize$
  - Repeat until you either find k or else you reach an empty cell in the table

Linear Probing: Delete

- Problem: don’t want to leave an empty space when deleting
- Option 1: when we delete, move the “last thing” with a matching hash to that location
- Option 2: “tombstone” deletion. When deleting something, leave a special marker to indicate something used to be there
-

Linear Probing: Delete

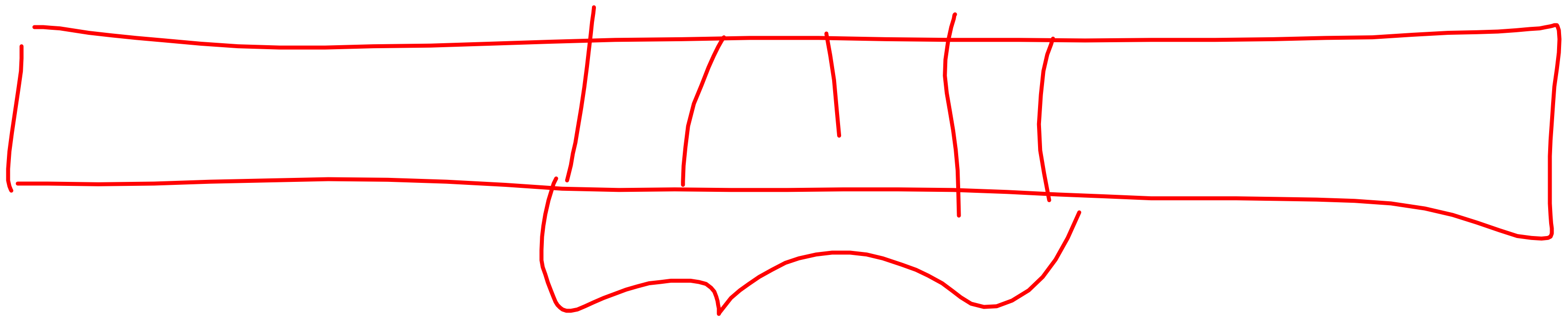
- Option 1: Find the last thing with a matching hash, move that into the spot you deleted from
- Option 2: Called "tombstone" deletion. Leave a special object that indicates an object was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item



Downsides of Linear Probing

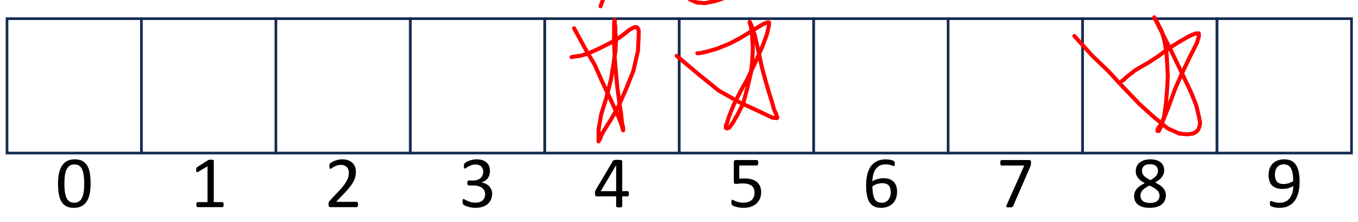
$\frac{n}{length}$

- What happens when λ approaches 1?
  - Longer and longer clusters of items
  - Runnings times get longer and longer



Quadratic Probing: Insert Procedure

- To insert k, v
  - Calculate $i = h(k)\,\%\,arrsize$
  - If $table[i]$ is occupied then try $(i + 1^2)\%\,arrsize$
  - If that is occupied try $(i + 2^2)\%\,arrsize$
  - If that is occupied try $(i + 3^2)\%\,arrsize$
  - If that is occupied try $(i + 4^2)\%\,arrsize$
  - …



Quadratic Probing: Example

- Insert:
  - 76 % 7 = 6
  - 40 % 7 = 5
  - 48 % 7 = 6
  - 5 → 5 + 1² → 5 + 2²
  - 55 → 6 + 1² → 6 + 2²
  - 47 → 5 + 1² → 5 + 2² → 5 + 3² → 5 + 4² → 5 + 5²

48		5	55		40	76
0	1	2	3	4	5	6

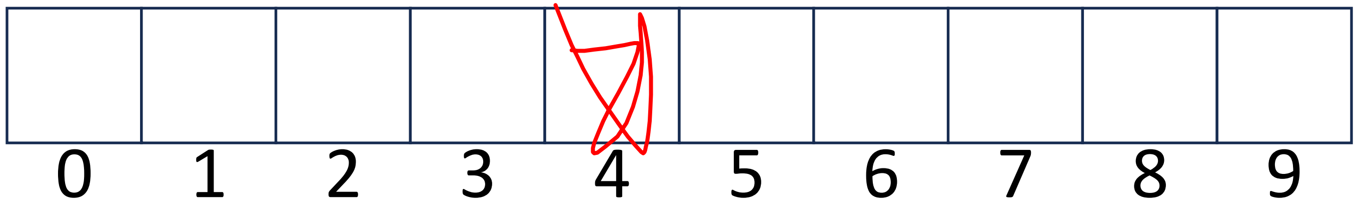
Using Quadratic Probing

- If you probe $tablesize$ times, you start repeating the same indices
- If $tablesize$ is prime and $\lambda < \frac{1}{2}$ then you're guaranteed to find an open spot in at most $tablesize/2$ probes

- Helps with the clustering problem of linear probing, but does not help if many things hash to the same value

Double Hashing: Insert Procedure

- Given h and g are both good hash functions
- To insert k, v
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied then try $\big(i + g(k)\big)\ \%\ size$
  - If that is occupied try $\big(i + 2 \cdot g(k)\big)\%\ size$
  - If that is occupied try $\big(i + 3 \cdot g(k)\big)\%\ size$
  - If that is occupied try $\big(i + 4 \cdot g(k)\big)\%\ size$
  - …



Rehashing

- If your load factor λ gets too large, copy everything over to a larger hash table
  - To do this: make a new array with a new hash function (maybe just a new modulus)
  - Re-insert all items into the new hash table with the new hash function
  - New hash table should be "roughly" double the size (but probably still want it to be prime)
- General Guideline:
  - Separate Chaining: rehash when $\lambda = 2$
  - Open Addressing: rehash when $\lambda = \frac{1}{2}$

Sorting

- Rearrangement of items into some defined sequence
  - Usually: reordering a list from smallest to largest according to some metric
- Why sort things?

- makes other things faster
  - Bin search
  - PQ
  - min/max/percentiles

More Formal Definition

$A[i] = 15$

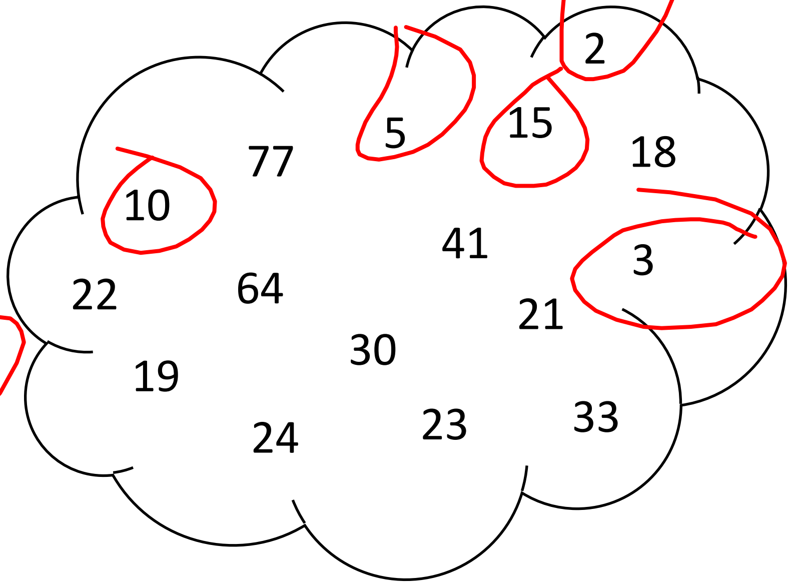
- Input:
  - An array A of items
  - A comparison function for these items
    - Given two items x and y, we can determine whether $x < y$, $x > y$, or $x = y$
- Output:
  - A permutation of A such that if $i \leq j$ then $A[i] \leq A[j]$
  - Permutation: a sequence of the same items but perhaps in a different order

Sorting "Landscape"

- There is no singular best algorithm for sorting
- Some are faster, some are slower
- Some use more memory, some use less
- Some are super extra fast if your data matches particular assumptions
- Some have other special properties that make them valuable
- No sorting algorithm can have only all the "best" attributes

"Moving Day" Sorting Algorithm

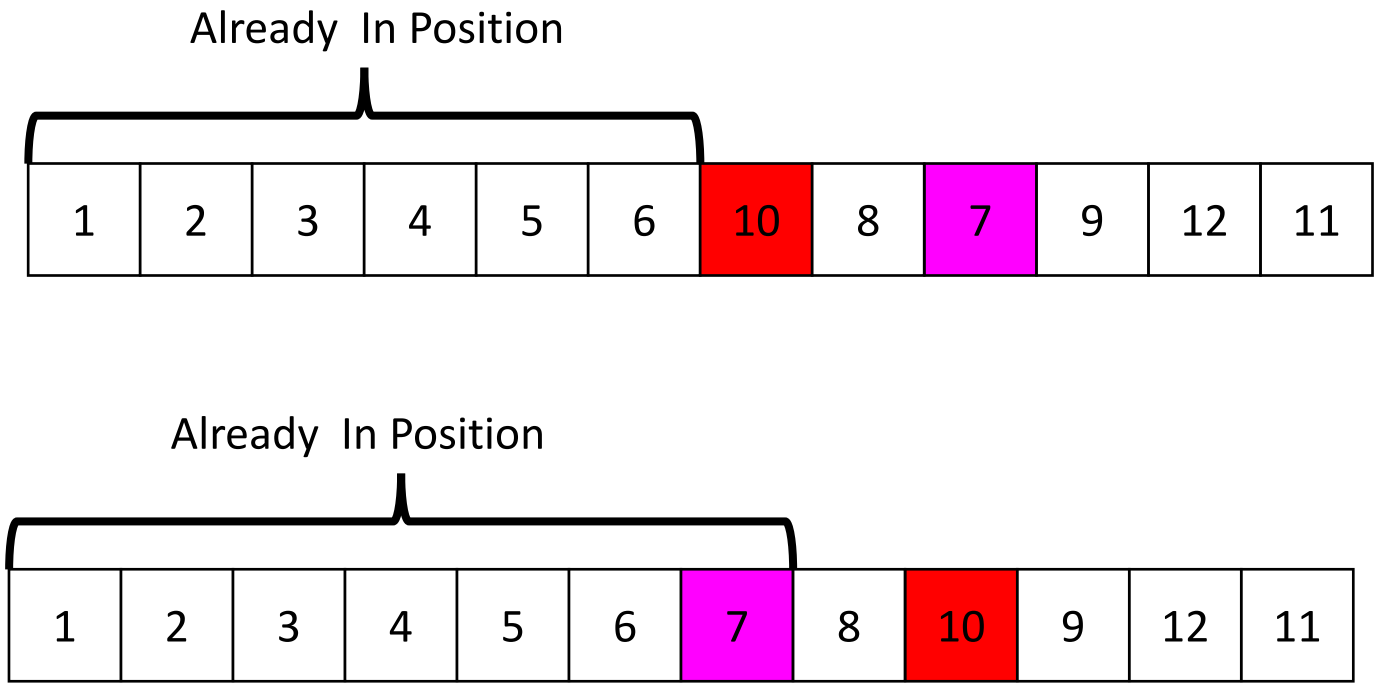
$\Theta(n^2)$



2	3	5	10	15											
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Selection Sort

- Idea: Find the next smallest element, swap it into the next index in the array



Selection Sort

- Swap the thing at index 0 with the smallest thing in the array
- Swap the thing at index 1 with the smallest thing after index 0
- …
- Swap the thing at index i with the smallest thing after index $i-1$

```
for (i=0; i<a.length; i++){
    smallest = i;
    for (j=i; j<a.length; j++){
        if (a[j]<a[smallest]){ smallest=j;}
    }
    temp = a[i];
    a[i] = a[smallest];
    a[smallest] = a[i];
}
```

Running Time:

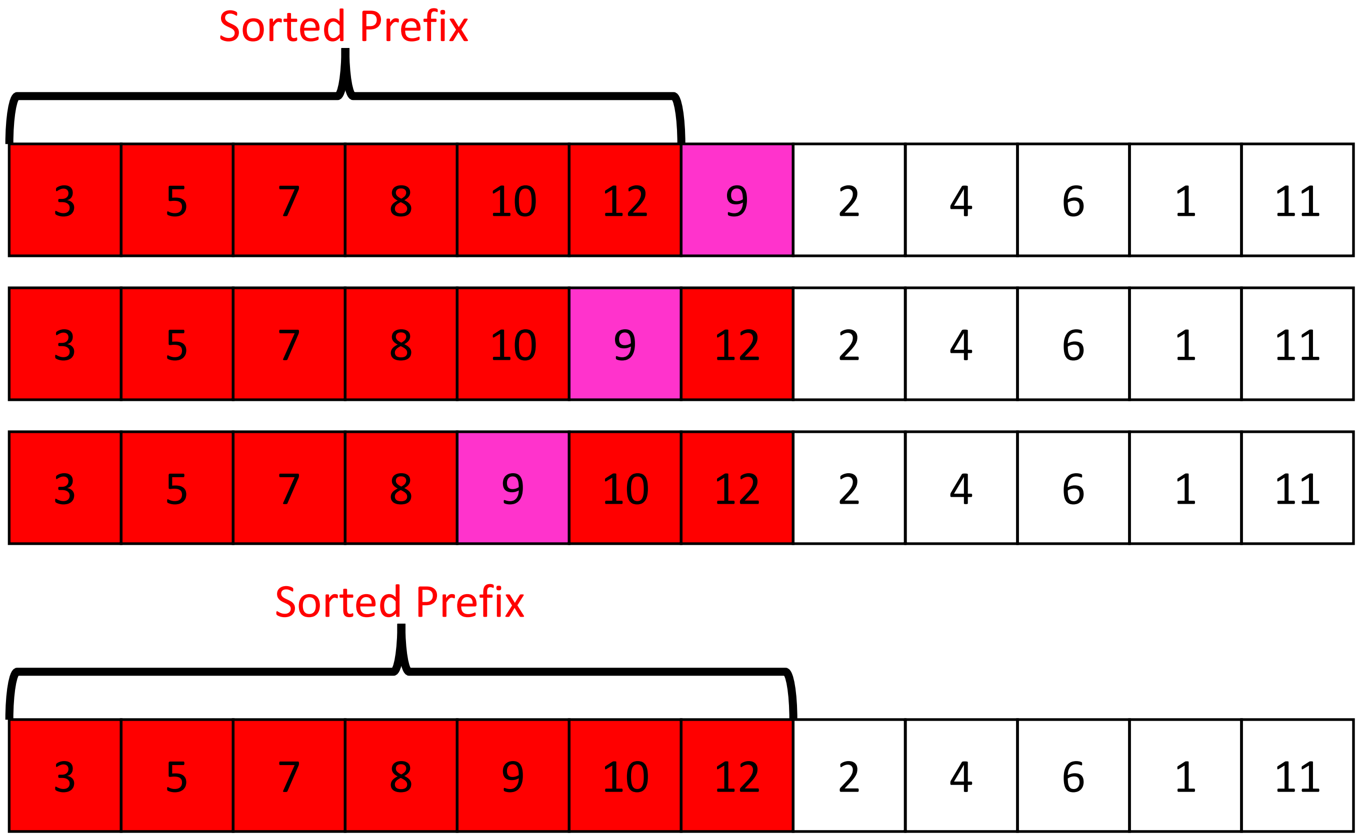
Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

10	77	5	15	2	22	64	41	18	19	30	21	3	24	23	33
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Insertion Sort

- Idea: Maintain a sorted list prefix, extend that prefix by "inserting" the next element



Insertion Sort

- If the items at index 0 and 1 are out of order, swap them
- Keep swapping the item at index 2 with the thing to its left as long as the left thing is larger
- …
- Keep swapping the item at index i with the thing to its left as long as the left thing is larger

```
for (i=1; i<a.length; i++){
    prev = i-1;
    while(a[i] < a[prev] && prev > -1){
        temp = a[i];
        a[i] = a[prev];
        a[prev] = a[i];
        i--;
        prev--;
    }
}
```

Running Time:

Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

10	77	5	15	2	22	64	41	18	19	30	21	3	24	23	33
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Aside: Bubble Sort – we won’t cover it

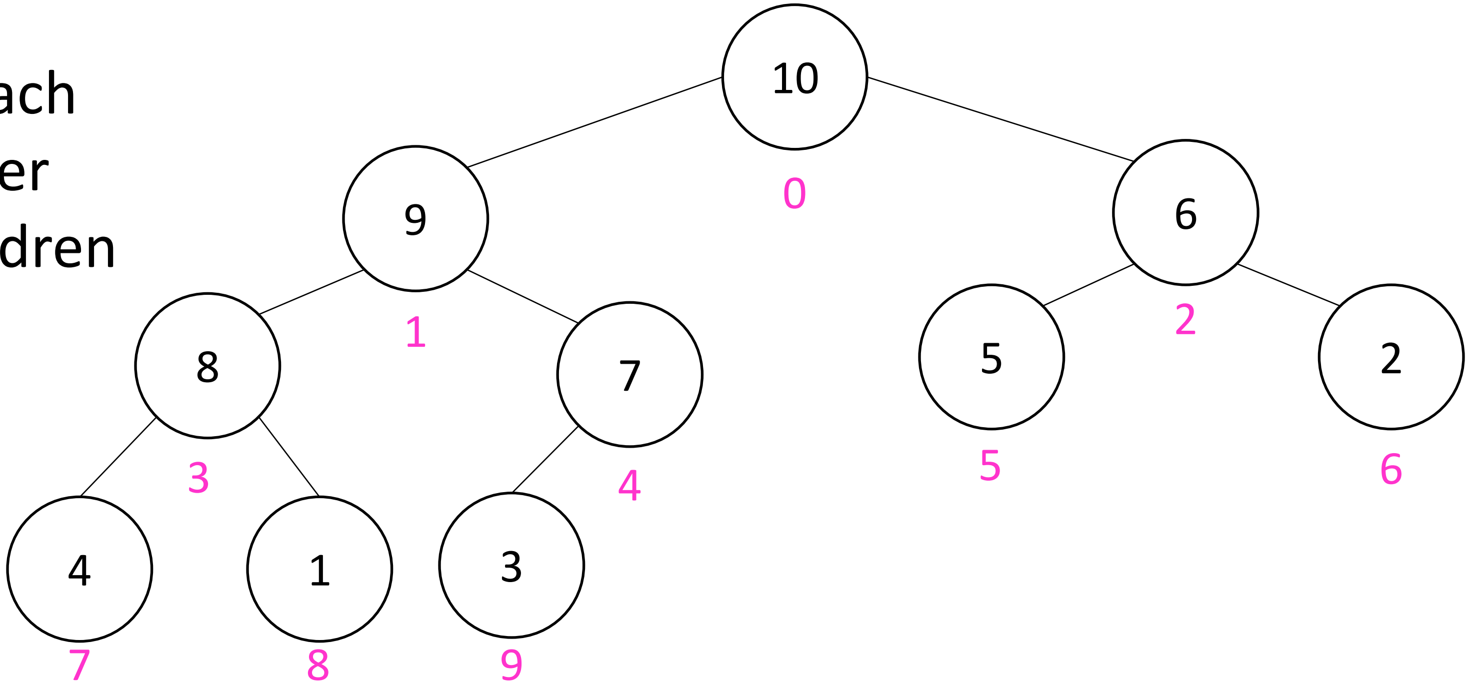
"the bubble sort seems to have nothing to recommend it, except a catchy name and the fact that it leads to some interesting theoretical problems” –Donald Knuth, The Art of Computer Programming

Heap Sort

- Idea: Build a maxHeap, repeatedly delete the max element from the heap to build sorted list Right-to-Left

10	9	6	8	7	5	2	4	1	3
0	1	2	3	4	5	6	7	8	9

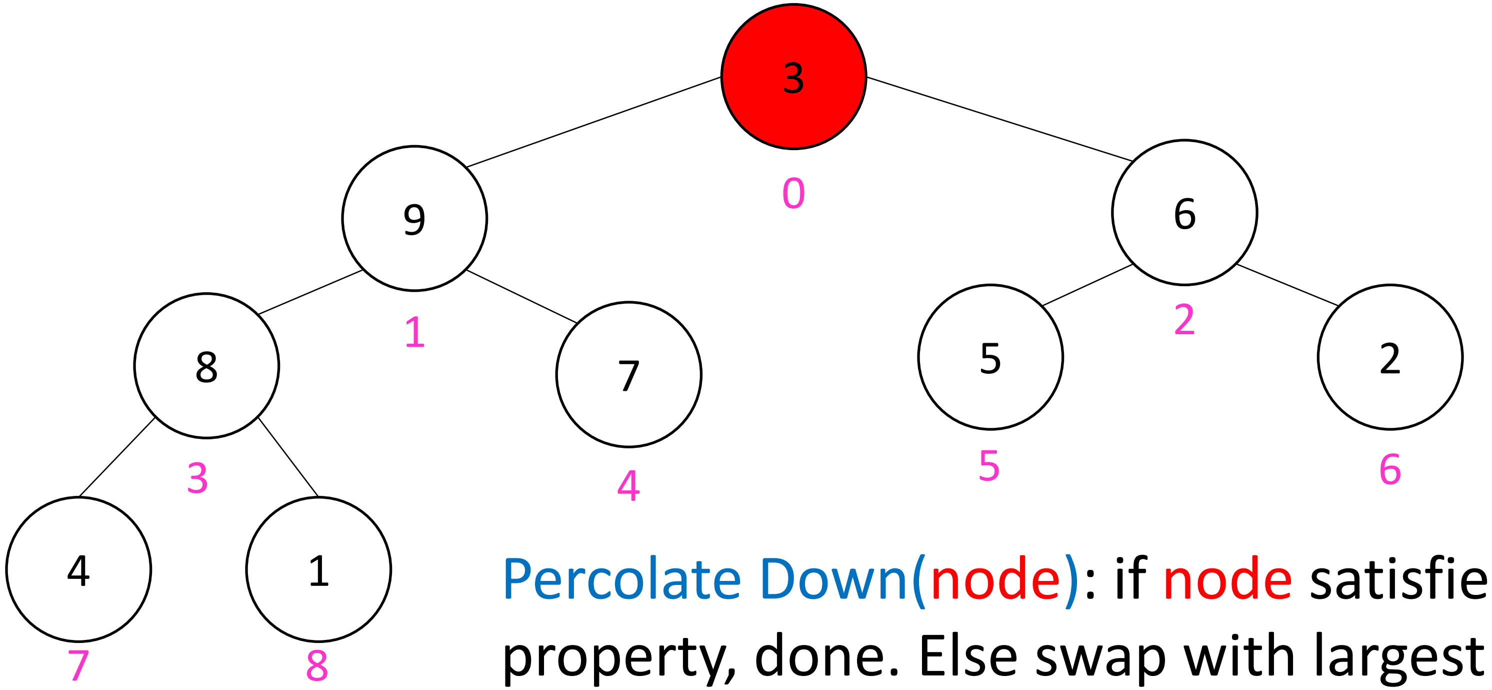
Max Heap Property: Each node is larger than its children



Heap Sort

- Remove the Max element (i.e. the root) from the Heap: replace with last element, call percolateDown(root)

3	9	6	8	7	5	2	4	1	
0	1	2	3	4	5	6	7	8	9



Percolate Down(node): if node satisfies heap property, done. Else swap with largest child and repeat on that subtree

Heap Sort

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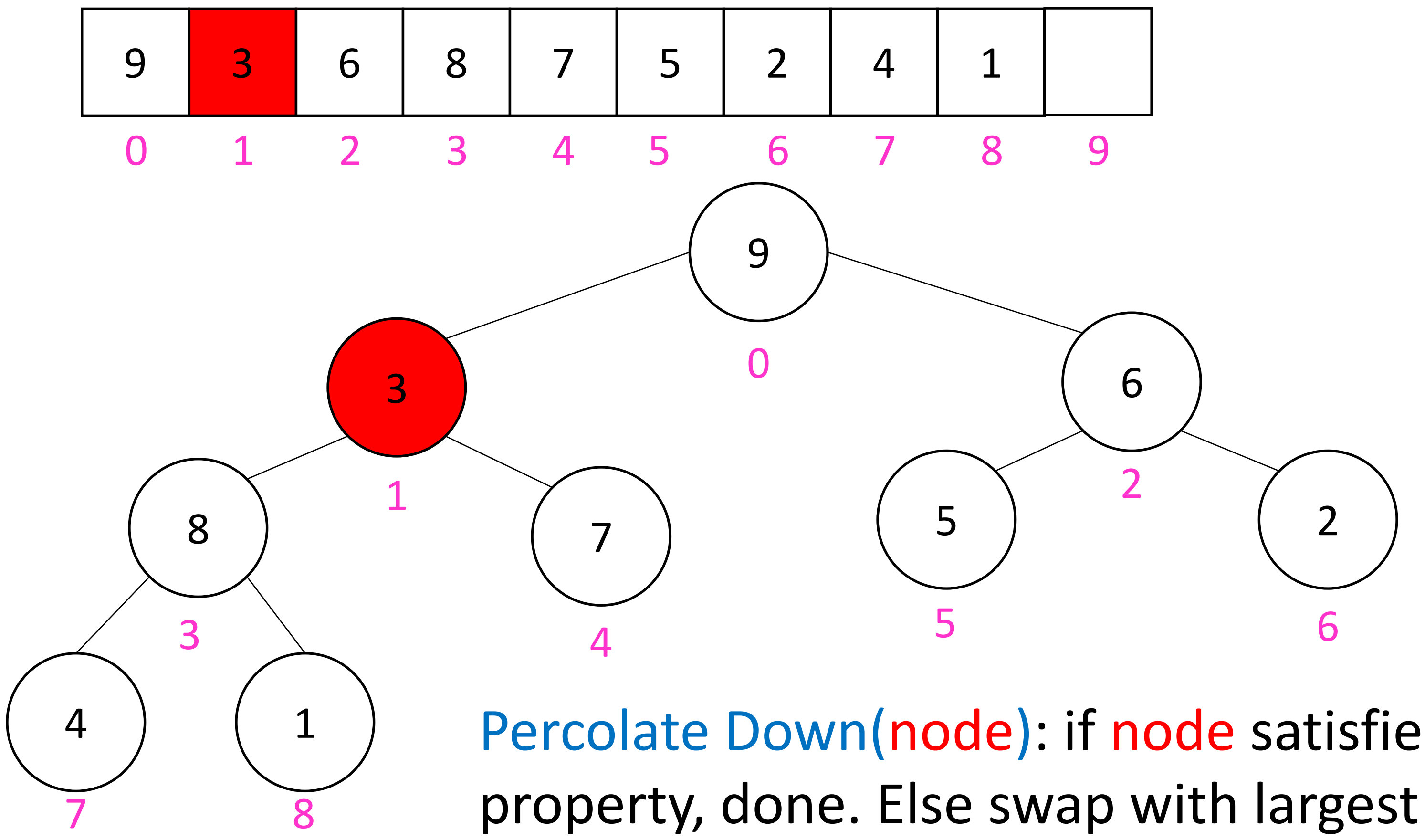

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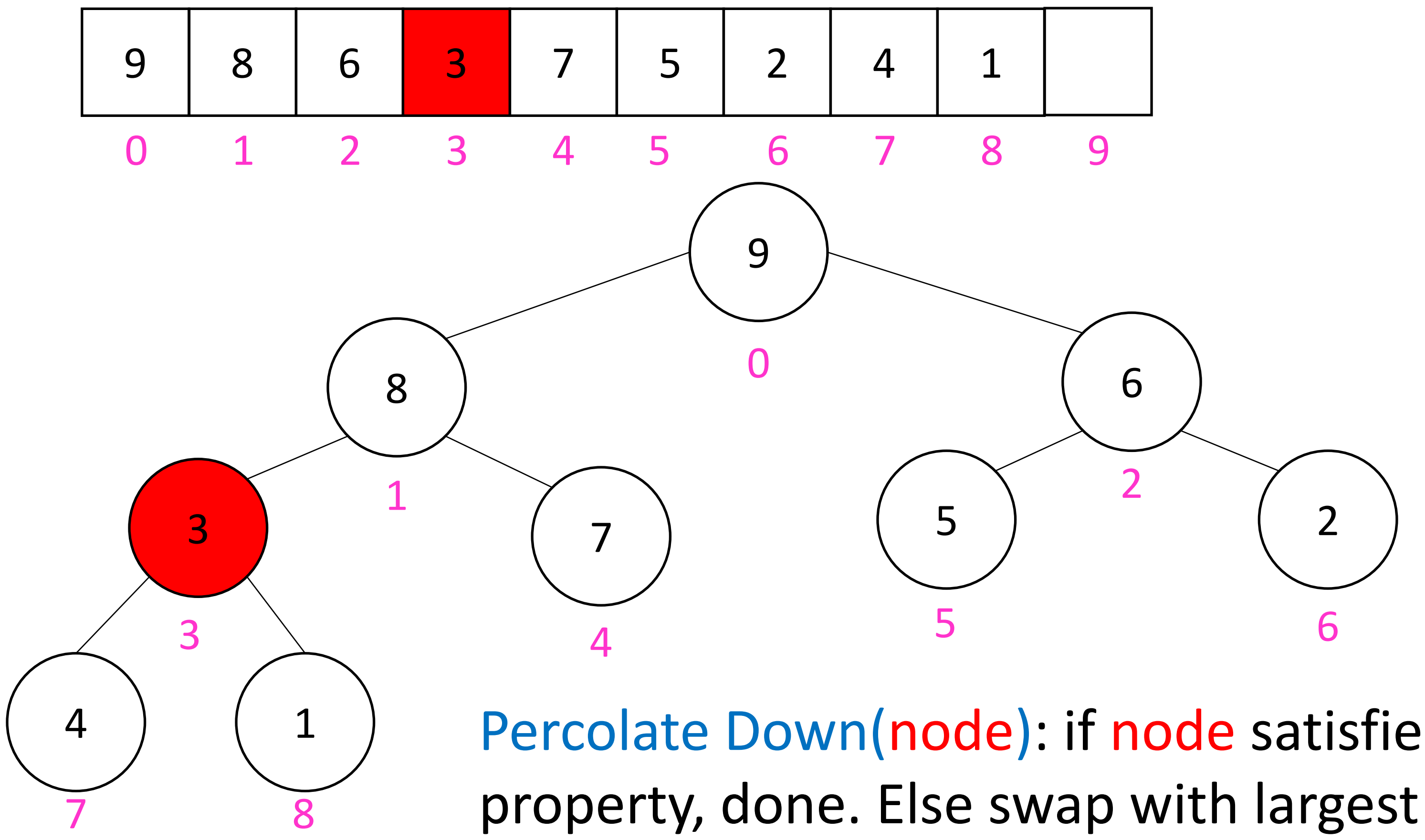

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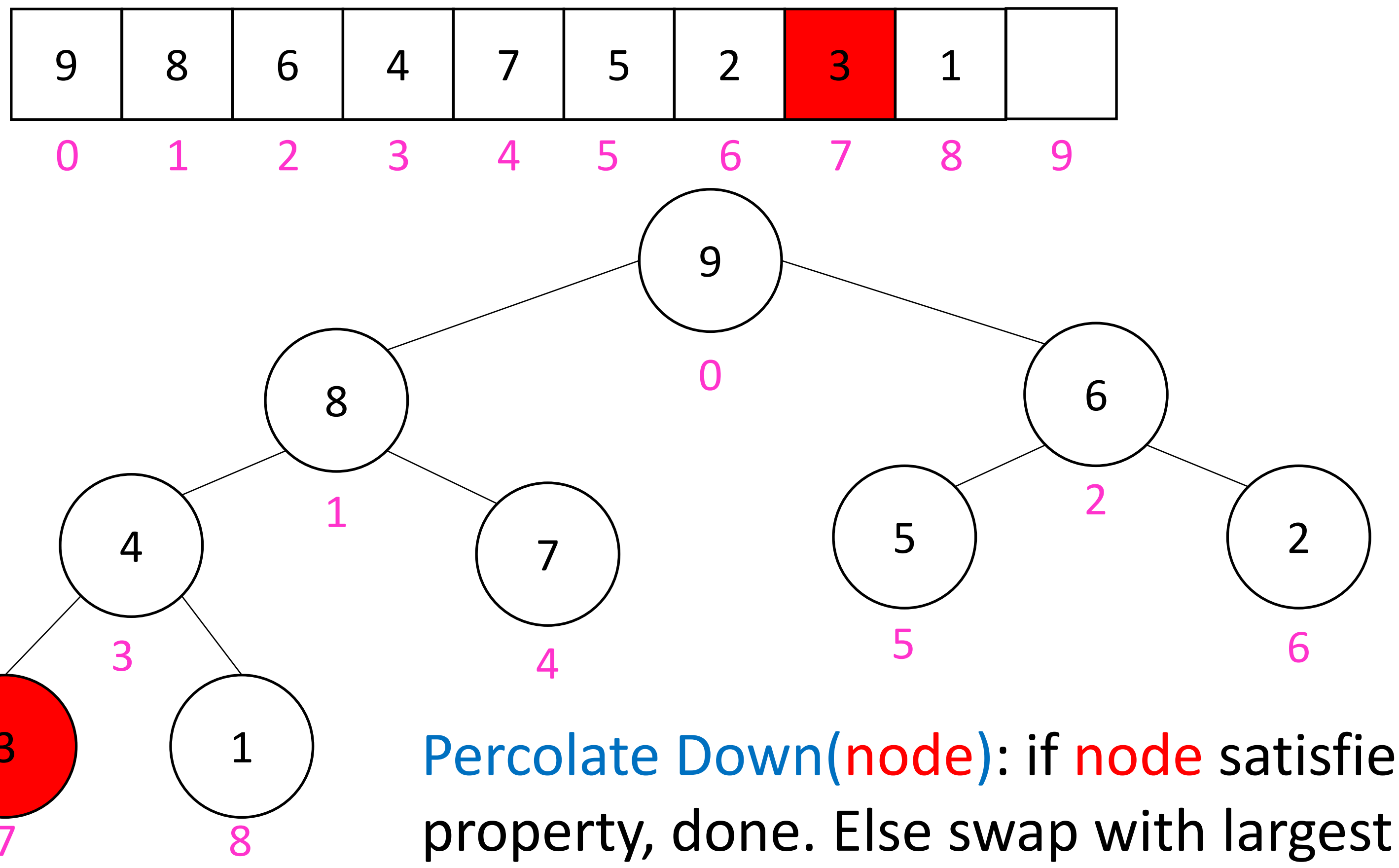


Percolate Down(node): if node satisfies heap property, done. Else swap with largest child and repeat on that subtree

Heap Sort

- Build a heap
- Call deleteMax
- Put that at the end of the array

```
myHeap = buildHeap(a);
for (int i = a.length-1; i>=0; i--){
	item = myHeap.deleteMax();
	a[i] = item;
}
```

Running Time:

Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

“In Place” Sorting Algorithm

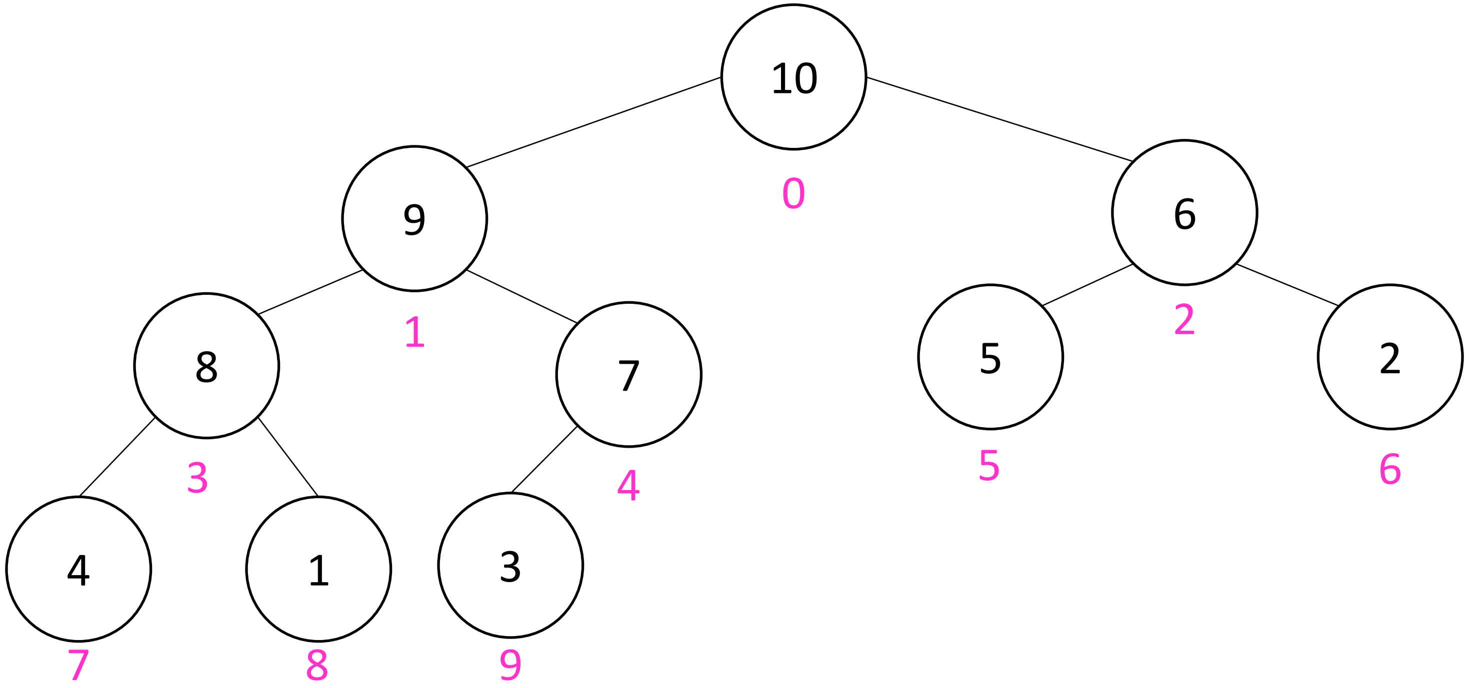
- A sorting algorithm which requires no extra data structures
- Idea: It sorts items just by swapping things in the same array given
- Definition: it only uses $\Theta(1)$ extra space

- Selection sort: In Place!
- Insertion sort: In Place!
- Heap sort: Not In Place!
    - But we can fix that!

In Place Heap Sort

- Idea: When “removing” an element from the heap, swap it with the last item of the heap then “pretend” the heap is one item shorter

10	9	6	8	7	5	2	4	1	3
0	1	2	3	4	5	6	7	8	9



Heap Sort

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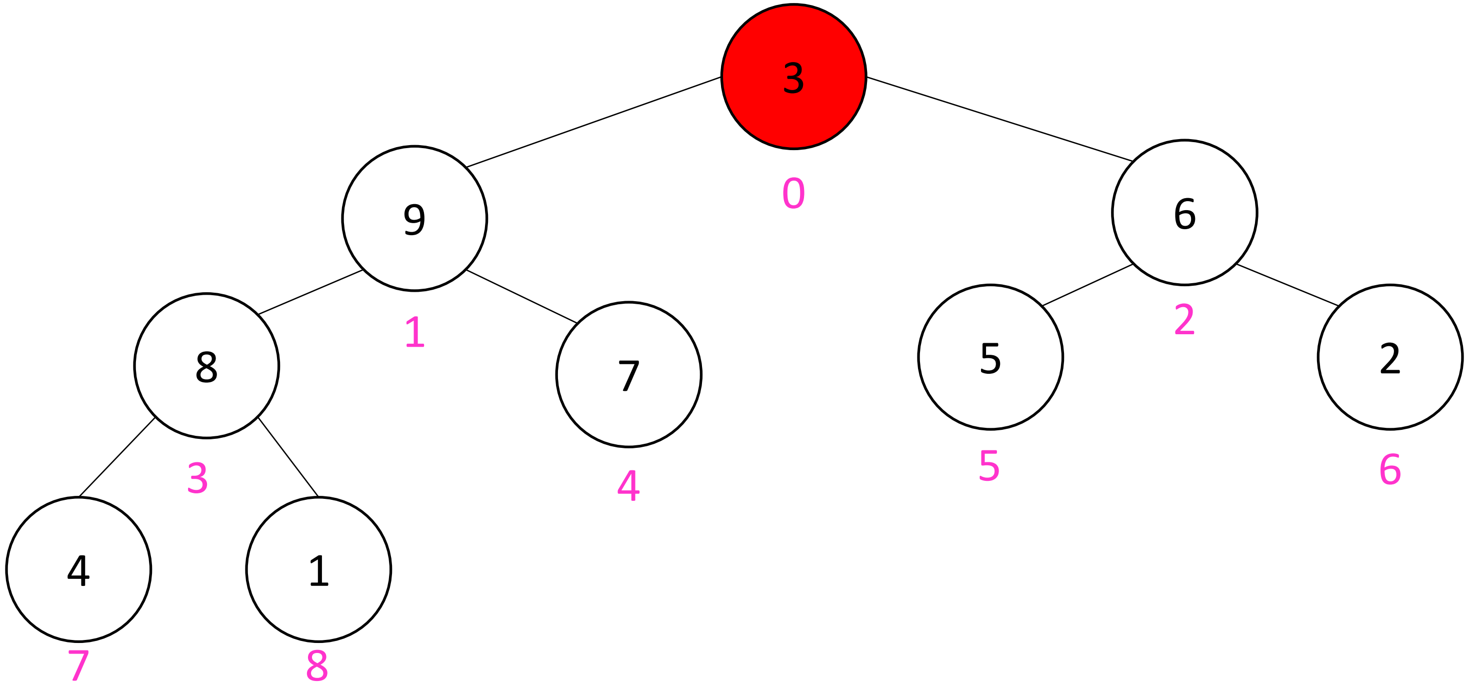
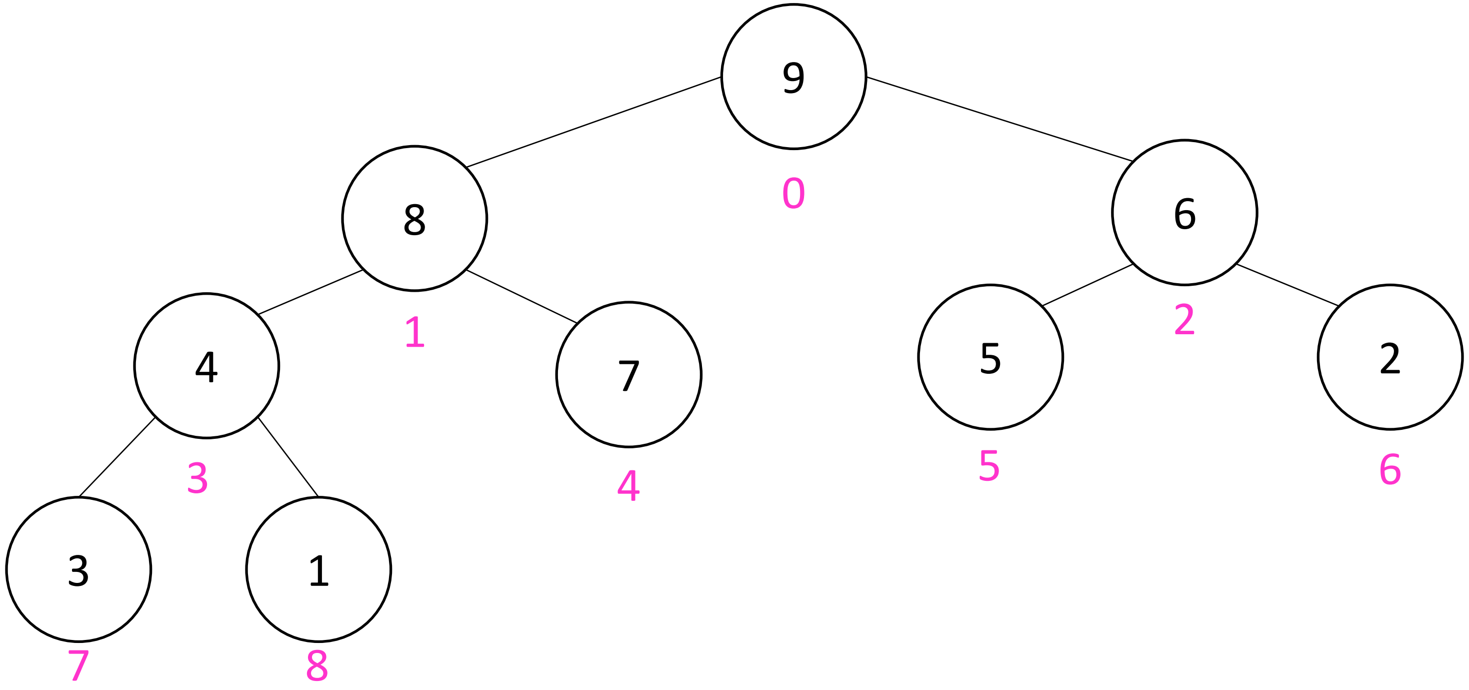
0	1	2	3	4	5	6	7	8	9



Heap Sort

- Idea: When “removing” an element from the heap, swap it with the last item of the heap then “pretend” the heap is one item shorter

9	8	6	4	7	5	2	3	1	10
0	1	2	3	4	5	6	7	8	9



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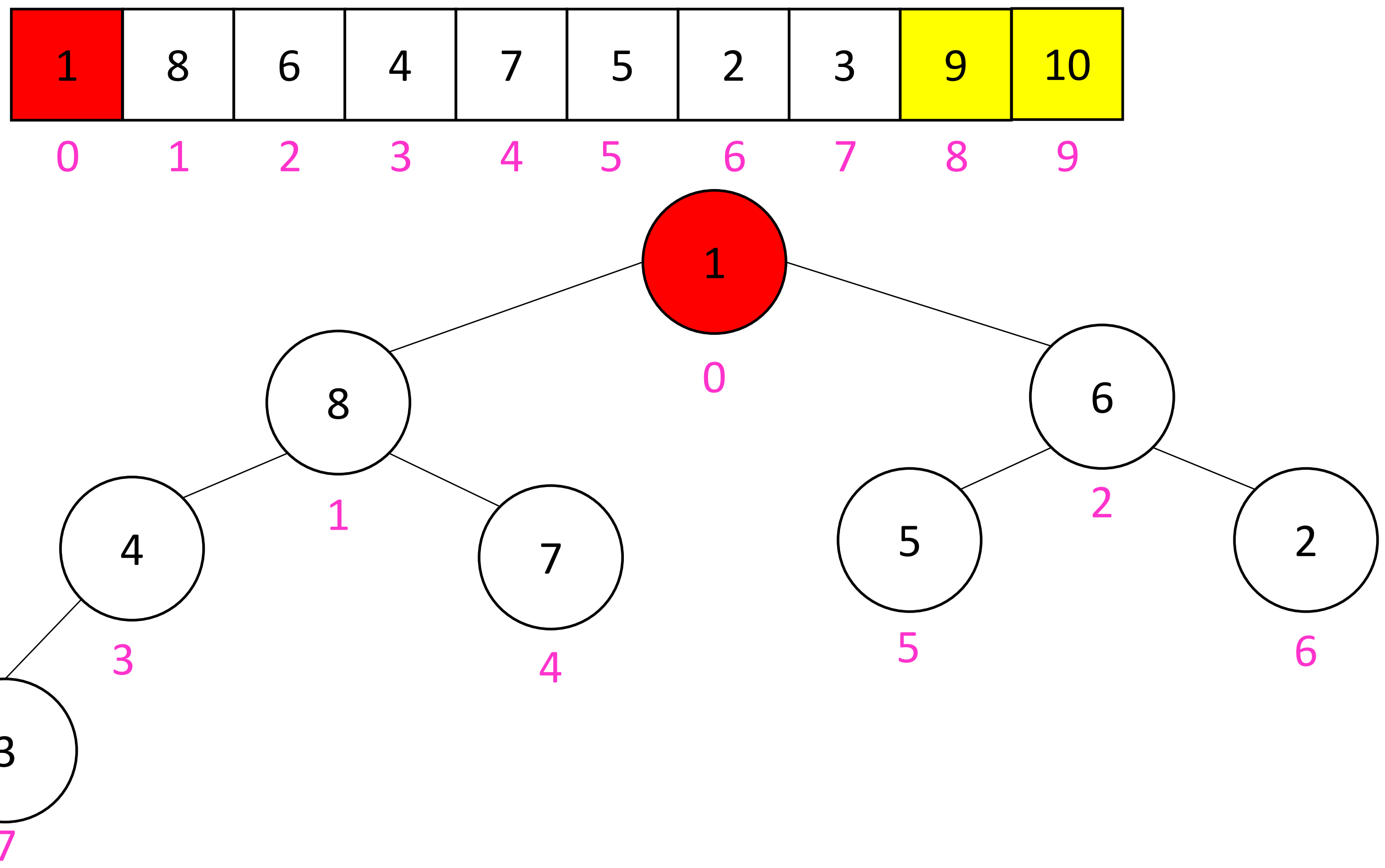
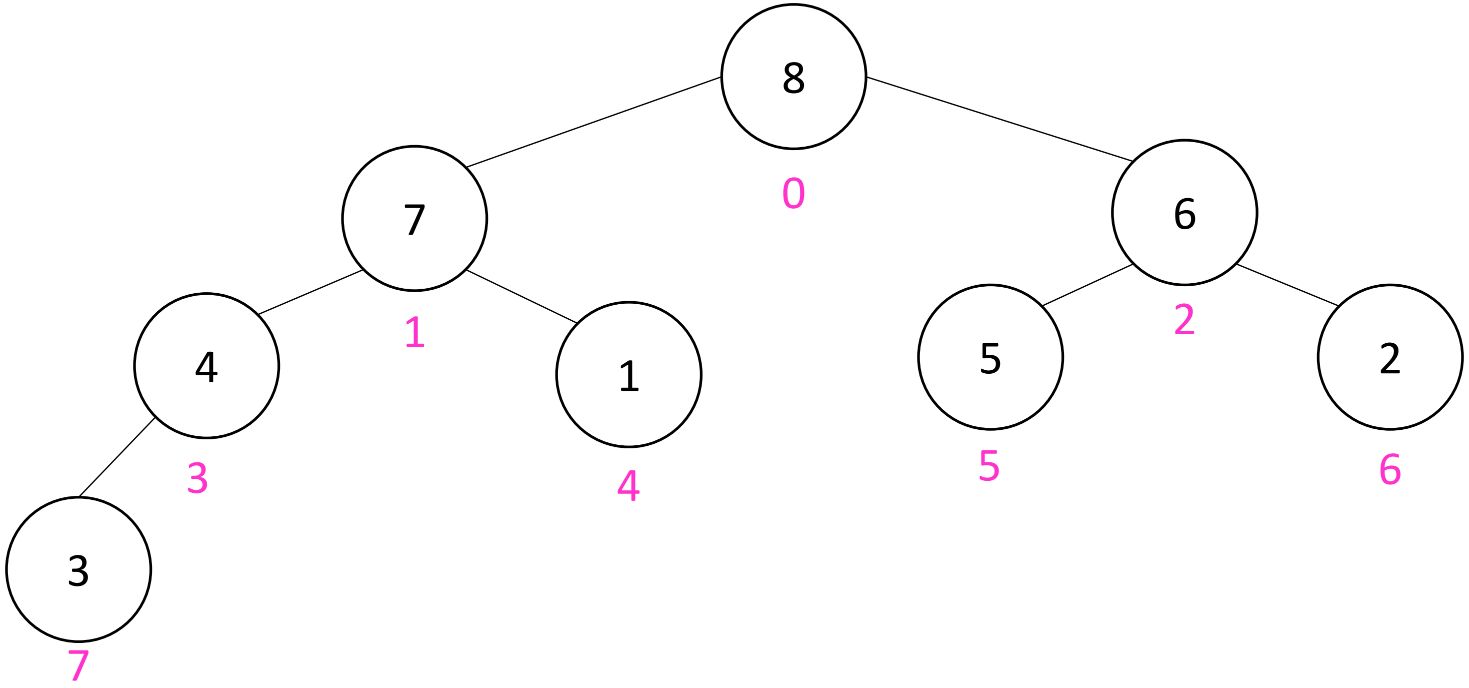


Heap Sort

- Idea: When “removing” an element from the heap, swap it with the last item of the heap then “pretend” the heap is one item shorter

1	8	6	4	7	5	2	3	9	10
0	1	2	3	4	5	6	7	8	9



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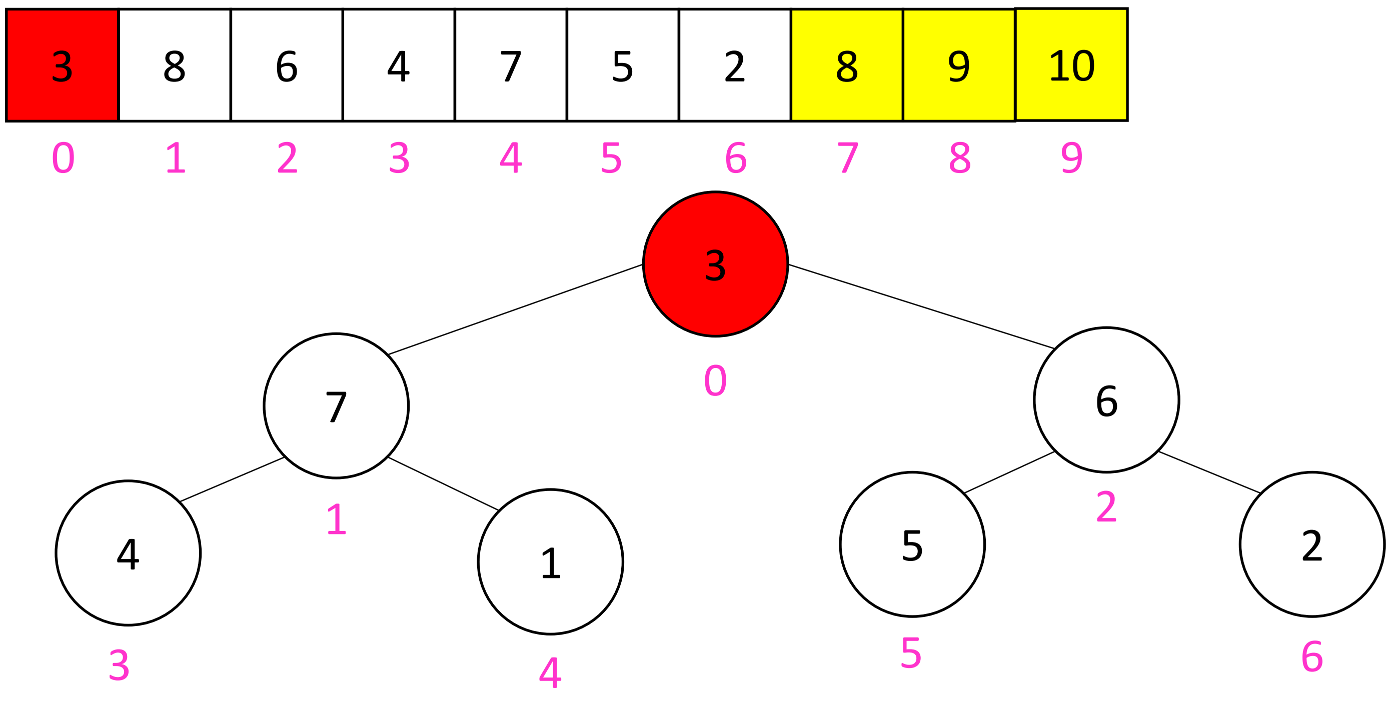
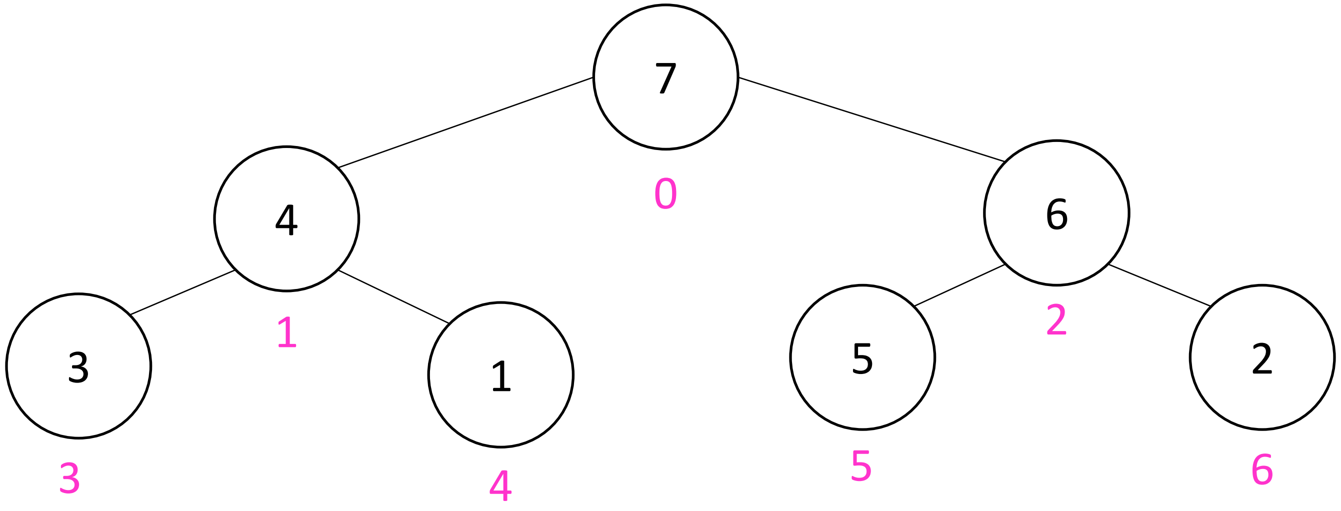


Heap Sort

- Idea: When "removing" an element from the heap, swap it with the last item of the heap then "pretend" the heap is one item shorter

0	1	2	3	4	5	6	7	8	9
3	8	6	4	7	5	2	8	9	10



In Place Heap Sort

- Build a heap using the same array (Floyd's build heap algorithm works)
- Call deleteMax
- Put that at the end of the array

```
buildHeap(a);
for (int i = a.length-1; i>=0; i--){
    temp=a[i]
    a[i] = a[0];
    a[0] = temp;
    percolateDown(0);
}
```

Running Time:

Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

Floyd’s buildHeap method

- Working towards the root, one row at a time, percolate down

```
buildHeap(){
    for(int i = size; i>0; i--){
        percolateDown(i);
    }
}
```