CSE 332 Autumn 2023
Lecture 22: ForkJoin Analysis

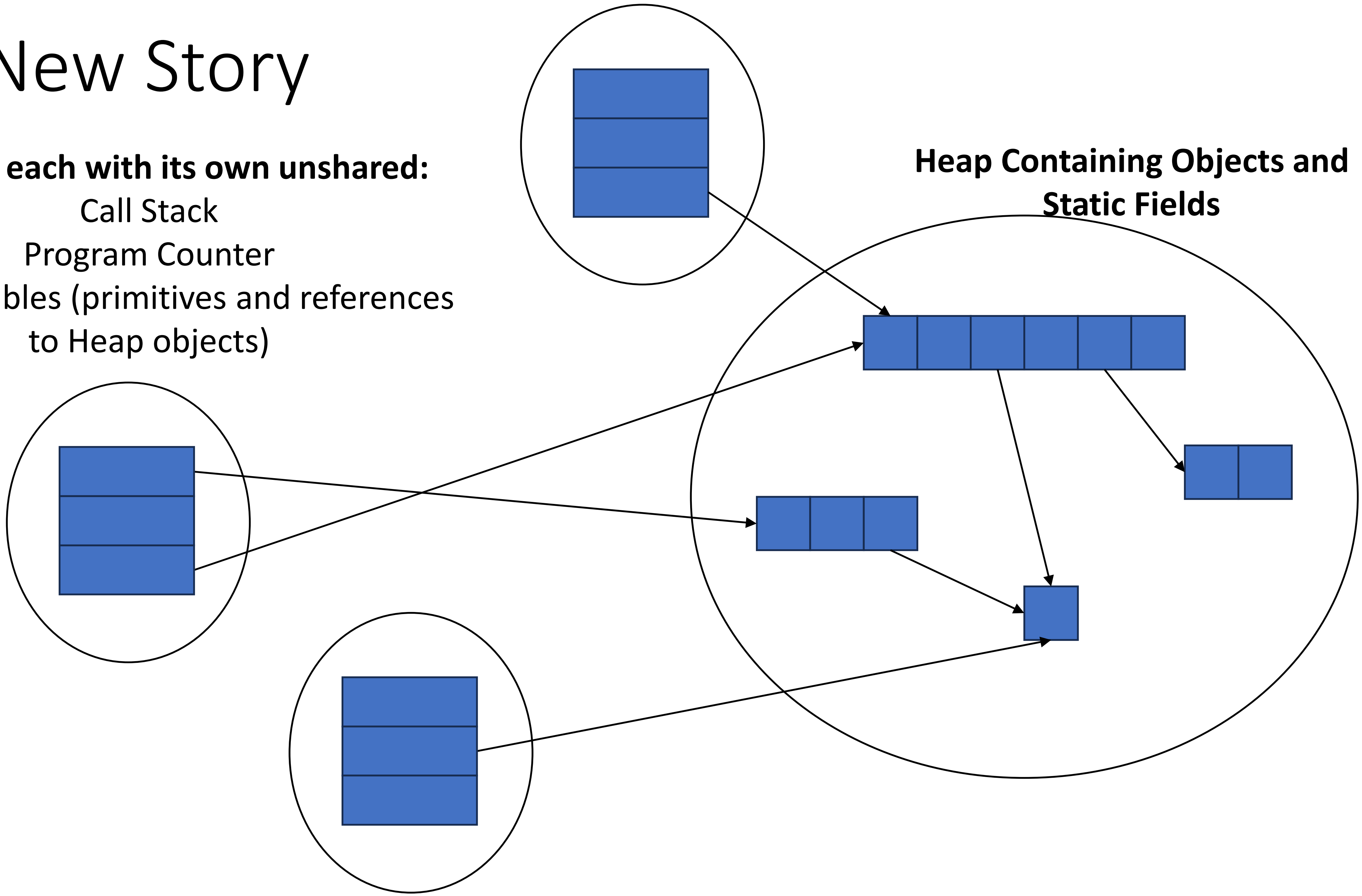
Nathan Brunelle

http://www.cs.uw.edu/332

New Story

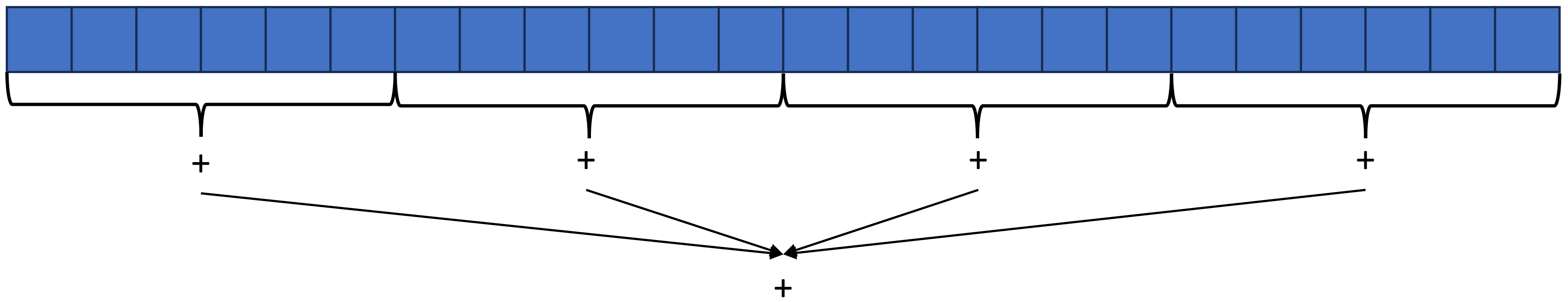
Threads, each with its own unshared:
Call Stack
Program Counter
Local Variables (primitives and references to Heap objects)

Heap Containing Objects and Static Fields

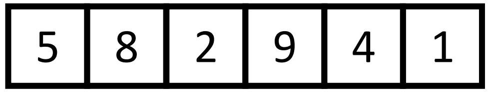


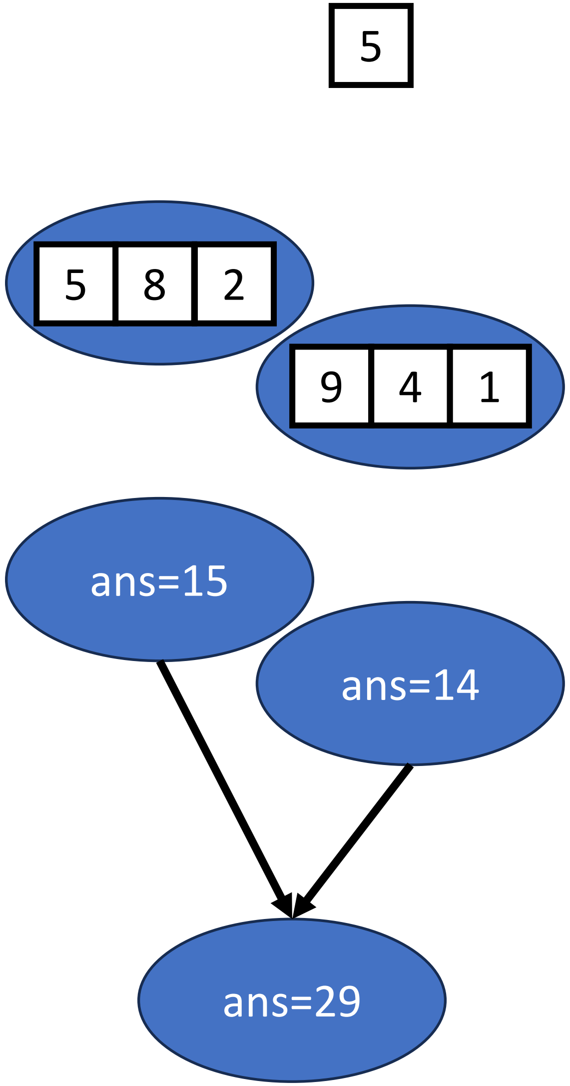
Back to Summing an Array

- Goal: Find the sum of an array
- Idea: 4 threads each find the sum of one quarter of the array
- Process:
  - Create 4 thread objects, each given a portion of the work
  - Call start() on each thread object to run it in parallel
  - Wait for threads to finish using join()
  - Add together their 4 answers for the final result



Parallel Sum





- **Base Case**:
  - If the list's length is smaller than the Sequential Cutoff, find the sum sequentially
- **Divide**:
  - Split the list into two "sublists" of (roughly) equal length, create a thread to sum each sublist.
- **Conquer**:
  - Call **start()** for each thread
- **Combine**:
  - Sum together the answers from each thread

Divide and Conquer with Threads

```
class SumThread extends java.lang.Thread {
	public void run(){ // override
		if(hi – lo < SEQUENTIAL_CUTOFF) // “base case”
			for(int i=lo; i < hi; i++) ans += arr[i];
		else {
			SumThread left = new SumThread(arr,lo,(hi+lo)/2); // divide
			SumThread right= new SumThread(arr,(hi+lo)/2,hi); // divide
			left.start(); // conquer
			right.start(); // conquer
			left.join(); // don’t move this up a line – why?
			right.join();
			ans = left.ans + right.ans; // combine
		}
	}
}
int sum(int[] arr){ // just make one thread!
	SumThread t = new SumThread(arr,0,arr.length);
	t.run();
	return t.ans; }
```

ForkJoin Framework

- This strategy is common enough that Java (and C++, and C#, and…) provides a library to do it for you!

What you would do in Threads	What to instead in ForkJoin
Subclass **Thread**	Subclass **RecursiveTask<V>**
Override **run**	Override **compute**
Store the answer in a field	Return a V from compute
Call **start**	Call **fork**
join synchronizes only	**join** synchronizes and returns the answer
Call **run** to execute sequentially	Call **compute** to execute sequentially
Have a topmost thread and call **run**	Create a pool and call **invoke**

Divide and Conquer with ForkJoin

```
class SumTask extends RecursiveTask<Integer> {
        int lo; int hi; int[] arr; // fields to know what to do
        SumTask(int[] a, int l, int h) { … }
        protected Integer compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        int ans = 0; // local var, not a field
                        for(int i=lo; i < hi; i++) {
                                ans += arr[i]; }
                        return ans;}
                else {
                        SumTask left = new SumTask(arr,lo,(hi+lo)/2); // divide
                        SumTask right= new SumTask(arr,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        int rightAns = right.compute(); //call compute directly (conquer)
                        int leftAns = left.join(); // get result from left
                        return leftAns + rightAns; // combine
                }
        }
}
```

Divide and Conquer with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();
int sum(int[] arr){
	SumTask task = new SumTask(arr,0,arr.length)
	return POOL.invoke(task); // invoke returns the value compute returns
}
```

Find Max with ForkJoin

```
class MaxTask extends RecursiveTask<Integer> {
        int lo; int hi; int[] arr; // fields to know what to do
        SumTask(int[] a, int l, int h) { … }
        protected Integer compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        int ans = Integer.MIN_VALUE; // local var, not a field
                        for(int i=lo; i < hi; i++) {
                                ans = Math.max(ans, arr[i]);}
                        return ans;
                else {
                        MaxTask left = new MaxTask(arr,lo,(hi+lo)/2); // divide
                        MaxTask right= new MaxTask(arr,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        int rightAns = right.compute(); //call compute directly (conquer)
                        int leftAns = left.join(); // get result from left
                        return Math.max(rightAns, leftAns); // combine
                }
        }
}
```

Other Problems that can be solved similarly

- Element Search
  - Is the value 17 in the array?
- Counting items with a certain property
  - How many elements of the array are divisible by 5?
- Checking if the array is sorted
- Find the smallest rectangle that covers all points in the array
- Find the first thing that satisfies a property
  - What is the leftmost item that is divisible by 20?

Reductions

- All examples of a category of computation called a reduction
  - We “reduce” all elements in an array to a single item
  - Requires operation done among elements is associative
    - $(x + y) + z = x + (y + z)$
  - The “single item” can itself be complex
    - E.g. create a histogram of results from an array of trials

Map

- Perform an operation on each item in an array to create a new array of the same size
- Examples:
  - Vector addition:
    - sum[i] = arr1[i] + arr2[i]
  - Function application:
    - out[i] = f(arr[i]);

Map with ForkJoin

```
class AddTask extends RecursiveAction {
        int lo; int hi; int[] arr; // fields to know what to do
        AddTask(int[] a, int[] b, int[] sum, int l, int h) { … }
        protected void compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        for(int i=lo; i < hi; i++) {
                                sum[i] = a[i] + b[i];}
                else {
                        AddTask left = new AddTask(a,b,sum,lo,(hi+lo)/2); // divide
                        AddTask right= new AddTask(a,b,sum,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        right.compute(); //call compute directly (conquer)
                        left.join(); // get result from left
                        return; // combine
                }
        }
}
```

Map with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();
Int[] add(int[] a, int[] b){
        ans = new int[a.length];
        AddTask task = new AddTask(a, b, ans, 0, a.length)
        POOL.invoke(task);
        return ans;
}
```

Maps and Reductions

- “Workhorse” constructs in parallel programming
- Many problems can be written in terms of maps and reductions
- With practice, writing them will become second nature
  - Like how over time for loops and if statements have gotten easier

Parallel Algorithm Analysis

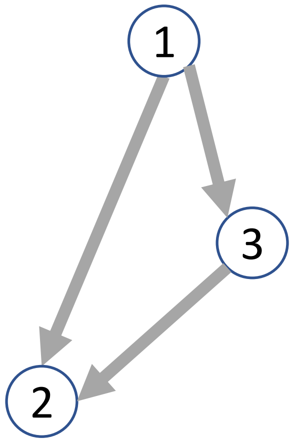
- How to define efficiency
  - Want asymptotic bounds
  - Want to analyze the algorithm without regard to a specific number of processors

Work and Span

- Let $T_P(n)$ be the running time if there are P processors available
- Two key measures of run time:
  - Work: How long it would take 1 processor, so $T_1(n)$
    - Just suppose all forks are done sequentially
    - Cumulative work all processors must complete
    - For array sum: $\Theta(n)$
  - Span: How long it would take an infinite number of processors, so $T_\infty(n)$
    - Theoretical ideal for parallelization
    - Longest "dependence chain" in the algorithm
    - Also called "critical path length" or "computation depth"
    - For array sum: $\Theta(\log n)$

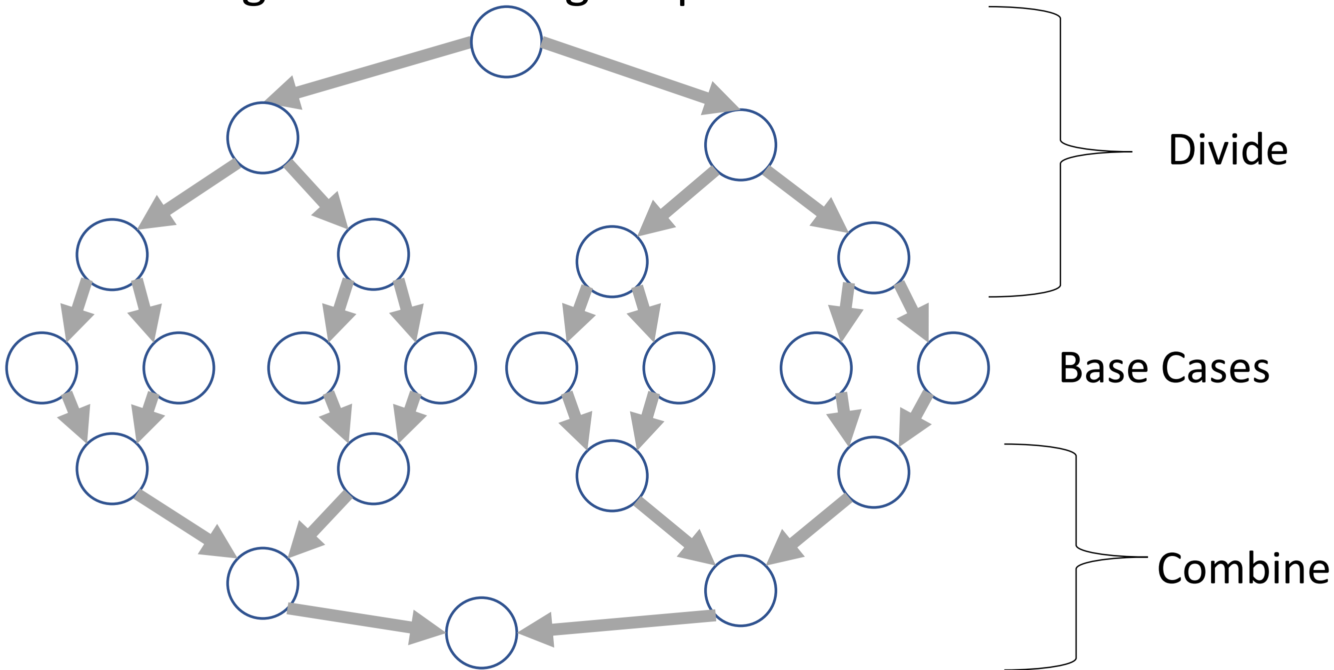
Directed Acyclic Graph (DAG)

- A directed graph that has no cycles
- Often used to depict dependencies
  - E.g. software dependencies, Java inheritance, dependencies among threads!



ForkJoin DAG

- Fork and Join each create a new node
  - Fork branches into two threads
    - Those two threads “depended on” their source thread to be created
  - Join combines to threads
    - The thread doing the combining “depends on” the other threads to finish



More Vocab

- Speed Up:
  - How much faster (than one processor) do we get for more processors
  - $T_1(n)/T_P(n)$
- Perfect linear Speedup
  - $\frac{T_1}{T_P} = P$
  - Hard to get in practice
  - “Holy Grail” or parallelizing
- Parallelism
  - Maximum possible speedup
  - T_1/T_∞
  - At some point more processors won’t be more helpful, when that point is depends on the span
- Writing parallel algorithms is about increasing span without substantially increasing work

Asymptotically Optimal T_P

- We know how to compute T_1 and T_∞, but what about T_P?
  - $T_P \geq \frac{T_1}{P}$
  - $T_P \geq T_\infty$
- An asymptotically optimal execution would be
  - $T_P(n) \in O\left(\frac{T_1(n)}{P} + T_\infty(n)\right)$
  - $T_1(n)/P$ dominates for small P, $T_\infty(n)$ dominates for large P
- ForkJoin Frameworks gives an expected time guarantee of asymptotically optimal!

Division of Responsibility

- Our job as ForkJoin Users:
  - Pick a good algorithm, write a program
  - When run, program creates a DAG of things to do
  - Make all the nodes a small-ish and approximately equal amount of work
- ForkJoin Framework Developer's job:
  - Assign work to available processors to avoid idling
    - Abstract away scheduling issues for the user
  - Keep constant factors low
  - Give the expected-time optimal guarantee

And now for some bad news…

- In practice it's common for your program to have:
  - Parts that parallelize well
    - Maps/reduces over arrays and other data structures
  - And parts that don't parallelize at all
    - Reading a linked list, getting input, or computations where each step needs the results of previous step
- These unparallelized parts can turn out to be a big bottleneck

Amdahl's Law (mostly bad news)

- Suppose $T_1 = 1$
  - Work for the entire program is 1
- Let S be the proportion of the program that cannot be parallelized
  - $T_1 = S + (1 - S) = 1$
- Suppose we get perfect linear speedup on the parallel portion
  - $T_P = S + \frac{1-S}{P}$
- For the entire program, the speed is:
  - $\frac{T_1}{T_P} = \frac{1}{S+\frac{1-S}{P}}$
- And so the parallelism (infinite processors) is:
  - $\frac{T_1}{T\_\infty} = \frac{1}{S}$

Ahmdal's Law Example

- Suppose 2/3 of your program is parallelizable, but 1/3 is not.
  - $S = \frac{2}{3}$
  - $T_1 = \frac{2}{3} + \frac{1}{3} = 1$
- $T_P = S + \frac{1-S}{P}$
- So if T_1 is 100 seconds:
  - $T_P = 33 + \frac{67}{P}$
  - $T_3 = 33 + \frac{67}{3} = 33 + 22 = 55$

Conclusion

- Even with many many processors the sequential part of your program becomes a bottleneck
- Parallelizable code requires skill and insight from the developer to recognize where parallelism is possible, and how to do it well.