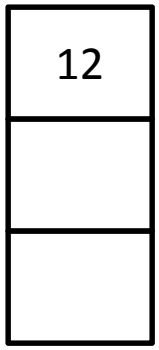
Solution for

- Section Worksheet 4, #7b & #7c
- Section Worksheet 4 #3a & #3b (same problem)

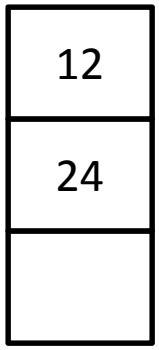
M=3, L=3

Insert 12



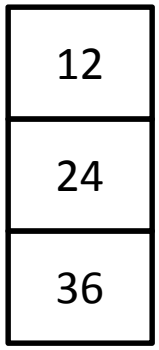
M=3, L=3

Insert 24



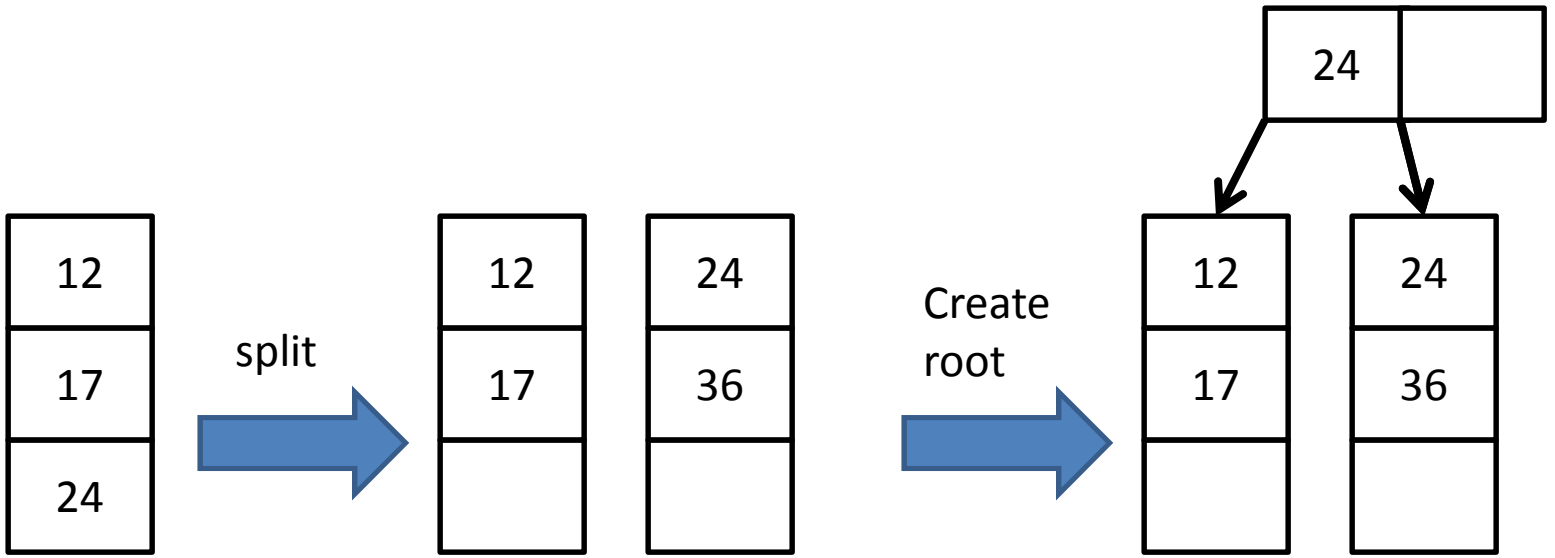
Insert 36

M=3, L=3



Insert 17

M=3, L=3



36

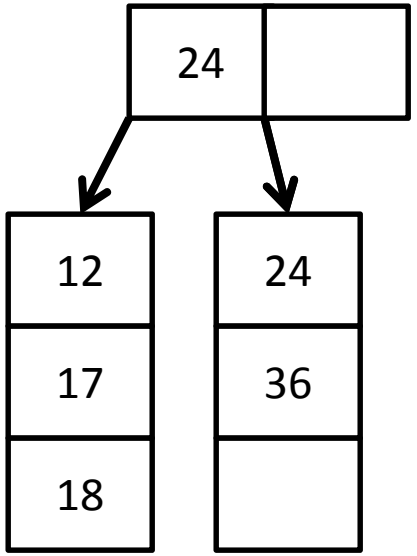
36 overflows!

Old node needs to have ceiling((L+1)/2) = 2 items, new node needs to have floor((L+1)/2) = 2 items.

First key in new root is the smallest item of right child

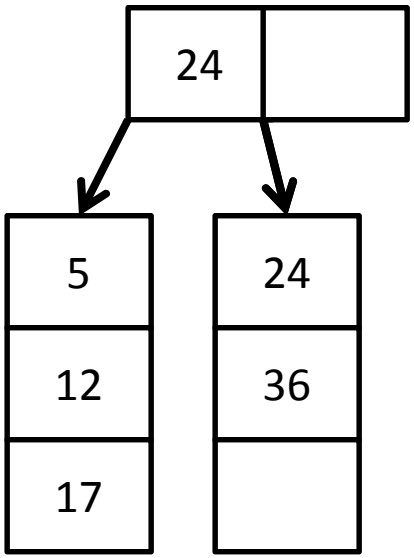
Insert 18

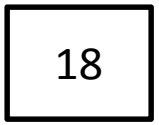
M=3, L=3



Insert 5

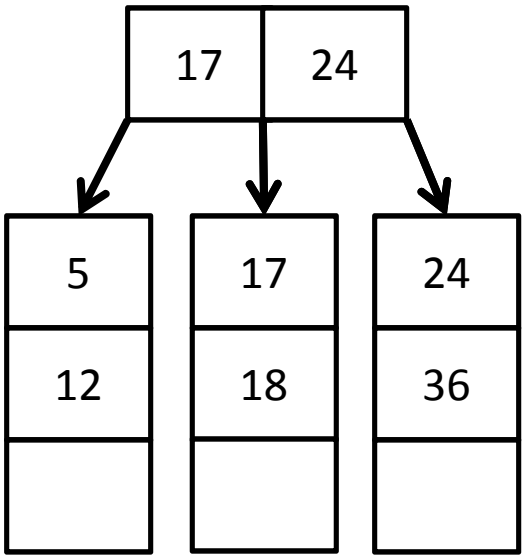
M=3, L=3





18 overflows!

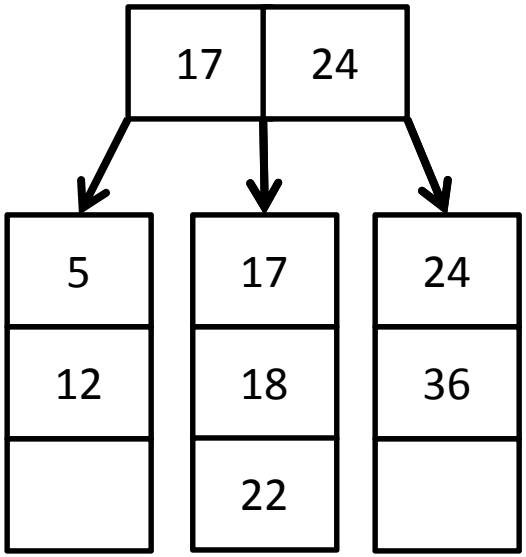
split



Old node needs to have ceiling((L+1)/2) = 2 items, new node needs to have floor((L+1)/2) = 2 items. Then fix pointer and internal node keys.

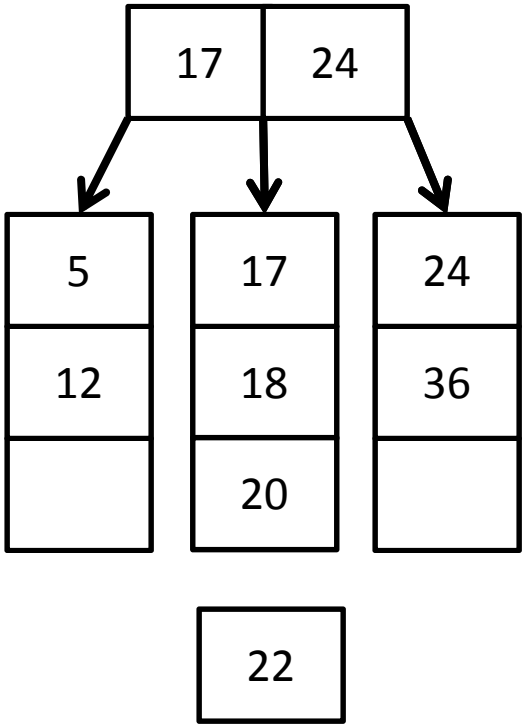
Insert 22

M=3, L=3



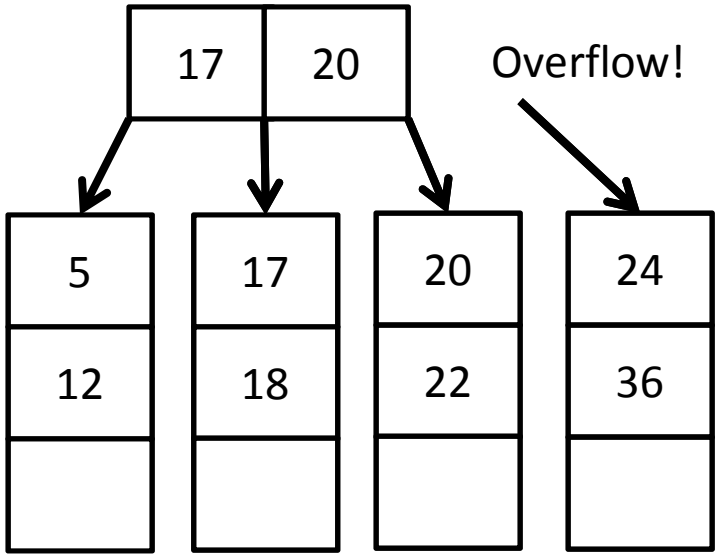
Insert 20 (step 1)

M=3, L=3



22

22 overflows!

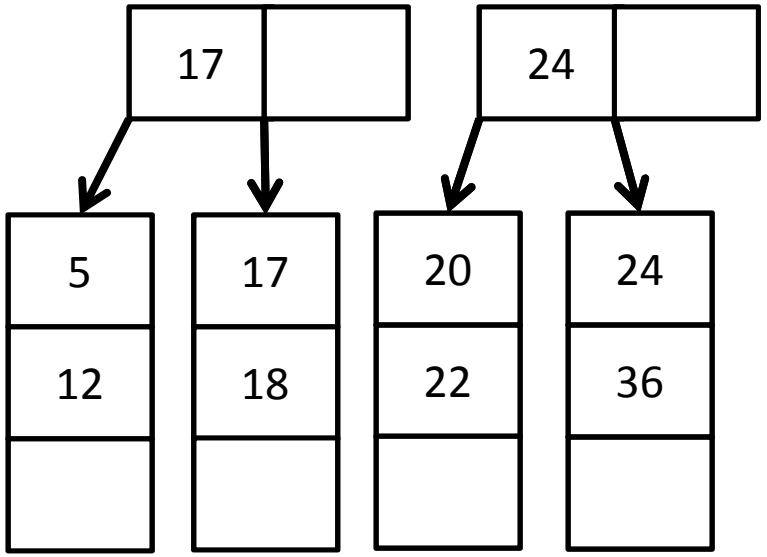


Split internal node

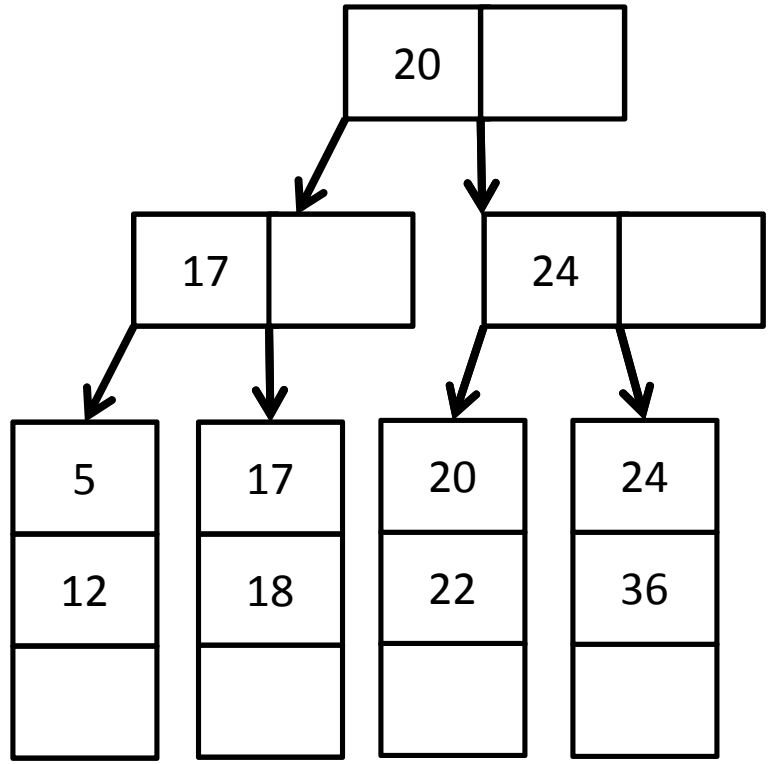
After splitting (17,18,20,22) into (17,18) & (20,22) we have overflow in the internal (parent) node

Insert 20 (step 2)

M=3, L=3



Create root

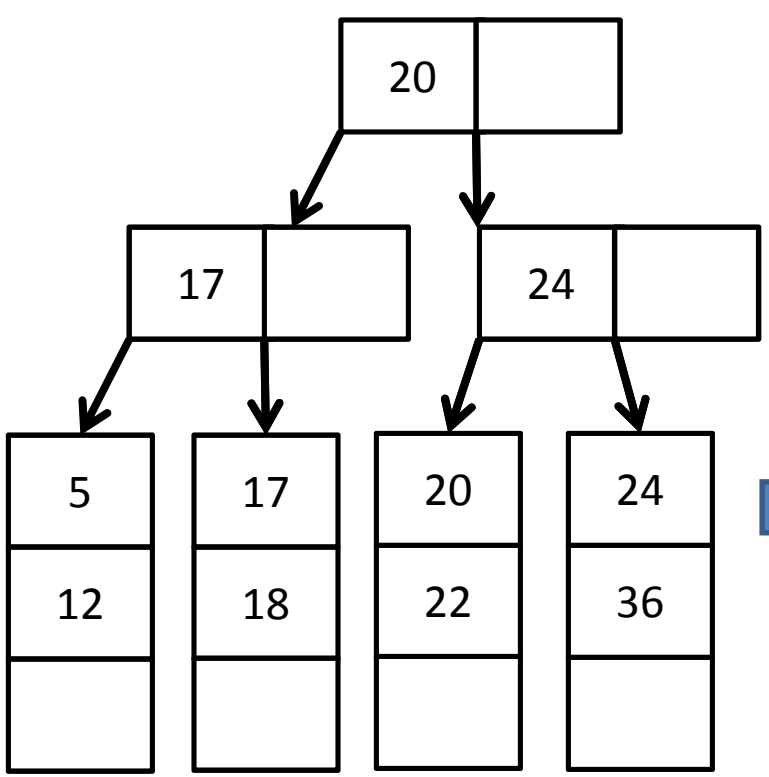


Old internal node needs to have ceiling((M+1)/2) = 2 pointers, new node needs to have floor((M+1)/2) = 2 pointers. This means they each have 1 key, since each key has a left and a right pointer.

Root's 1st key should be the smallest item in its right child, so smallest item of (20,22,24,36) is 20.

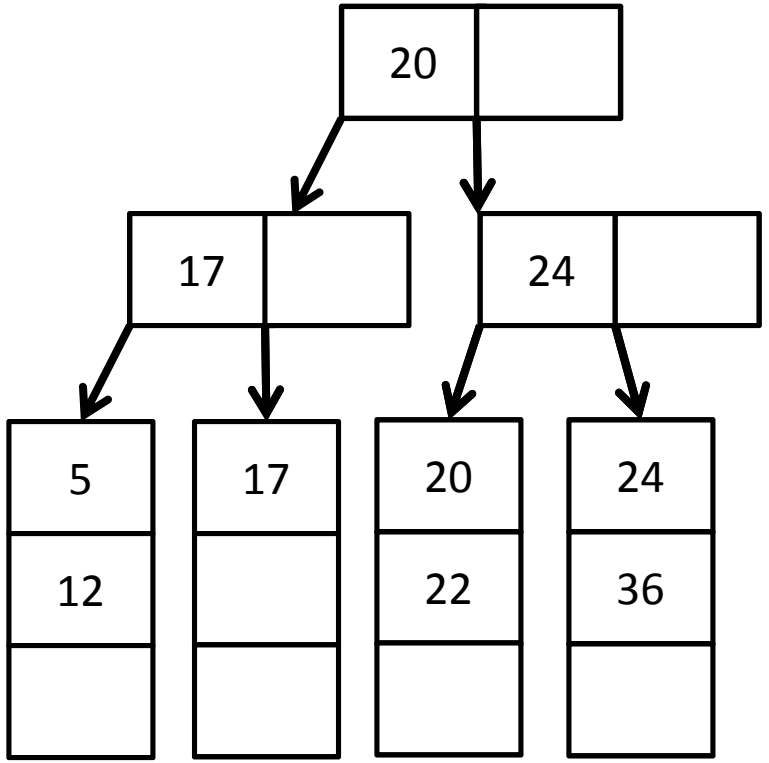
Delete 17 (step 1)

M=3, L=3



Original Tree

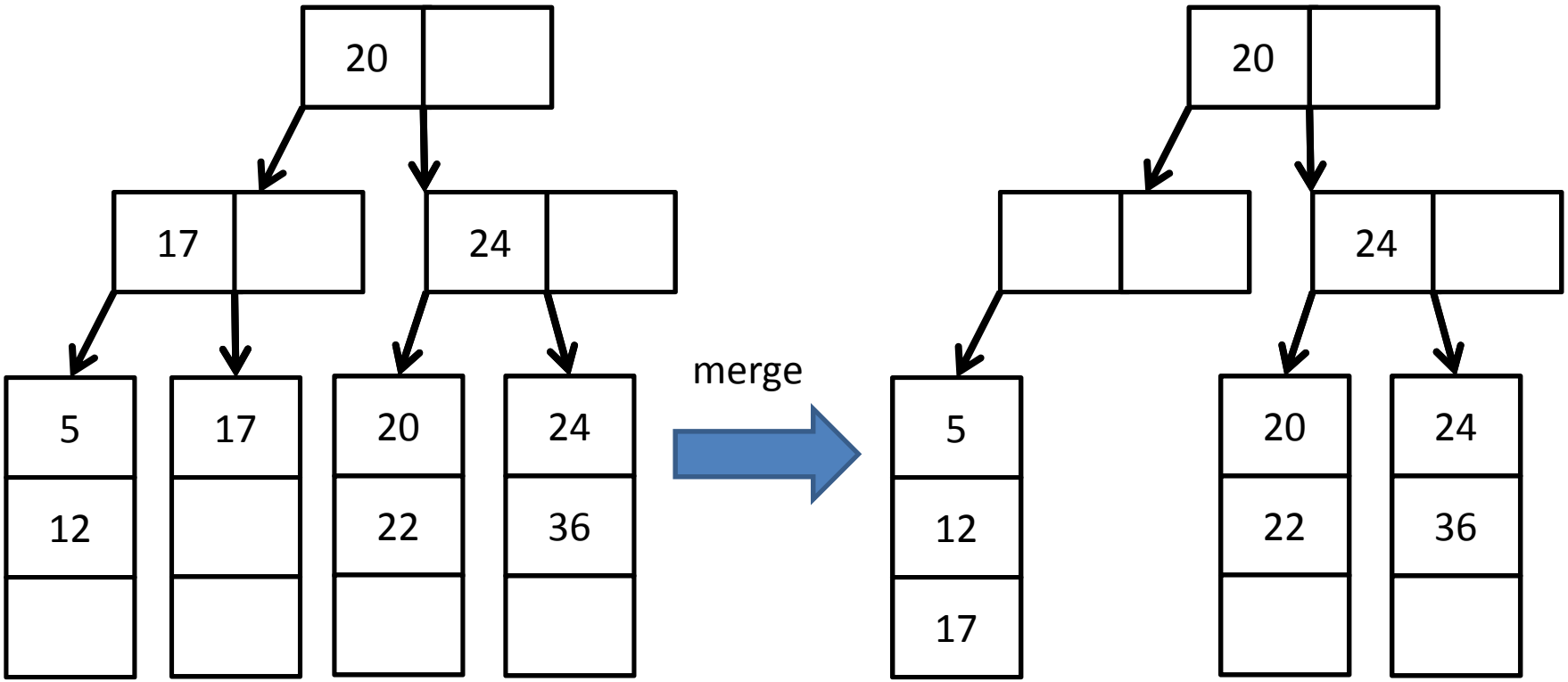
Delete 17



Underflow, because leaf now has ceiling(L/2) - 1 items in leaf node.

Delete 17 (step 2)

M=3, L=3

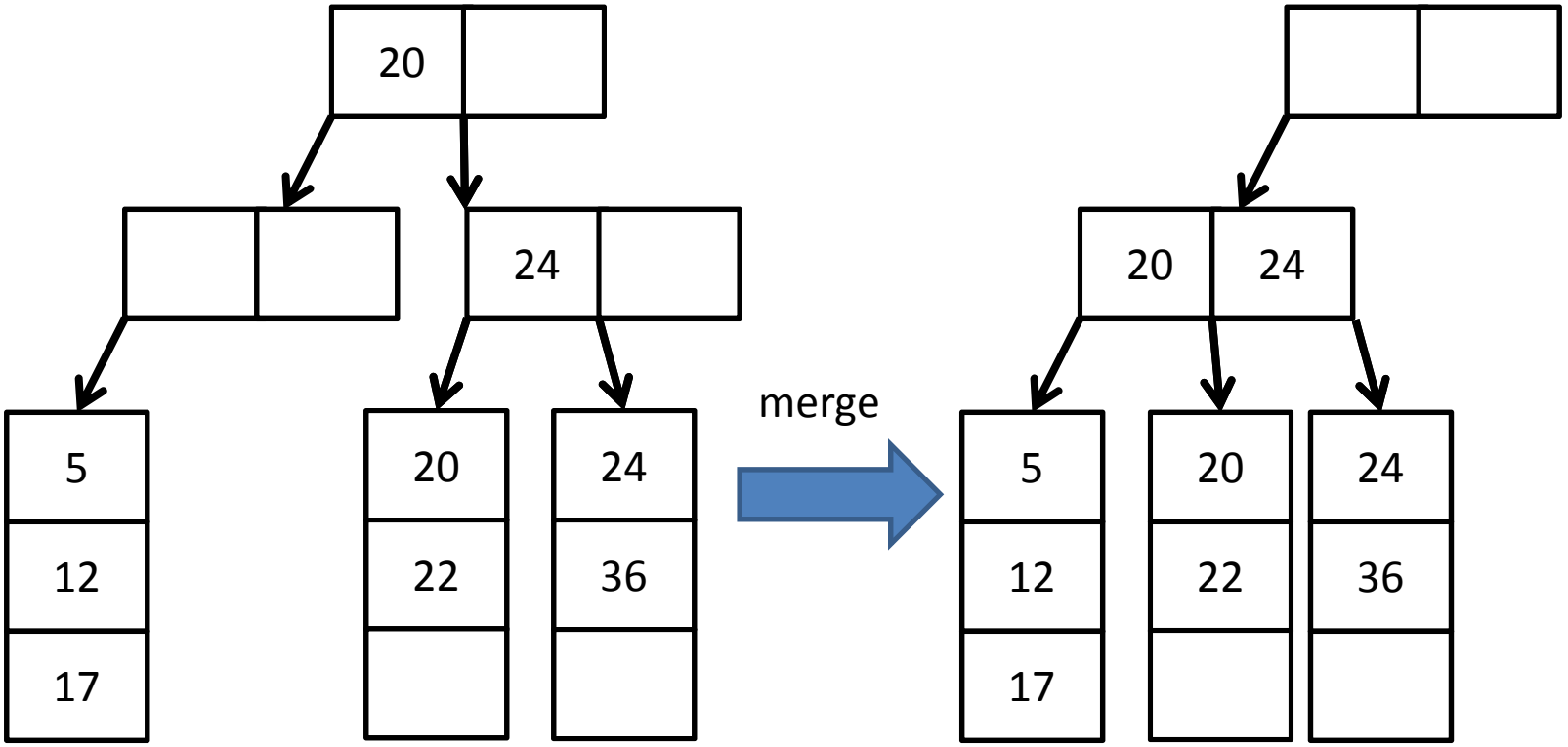


If (17)'s neighbor, (5,12), has enough items such that it won't underflow, we could adopt 12, but that would cause underflow, so need to merge

Merging leaf nodes caused underflow in parent internal node, which now only ceiling(M/2)-1 pointers (i.e. only 1 pointer). So need to fix!

Delete 17 (step 3)

M=3, L=3

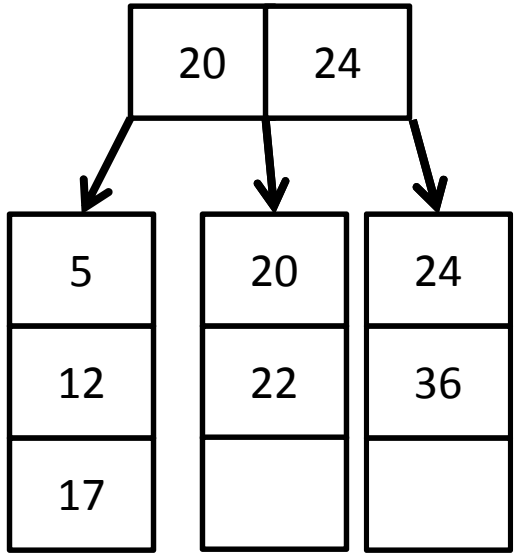


Internal node's neighbor, (24), does not have enough pointers such that it will not underflow if we adopt one, so we must merge these two internal nodes.

But now, the root is underflowed (only 1 pointer), so…

Delete 17 (step 4)

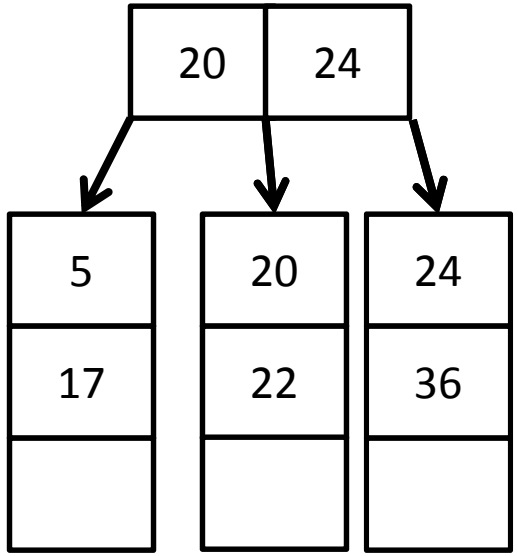
M=3, L=3



….we can just delete the root.

Delete 12

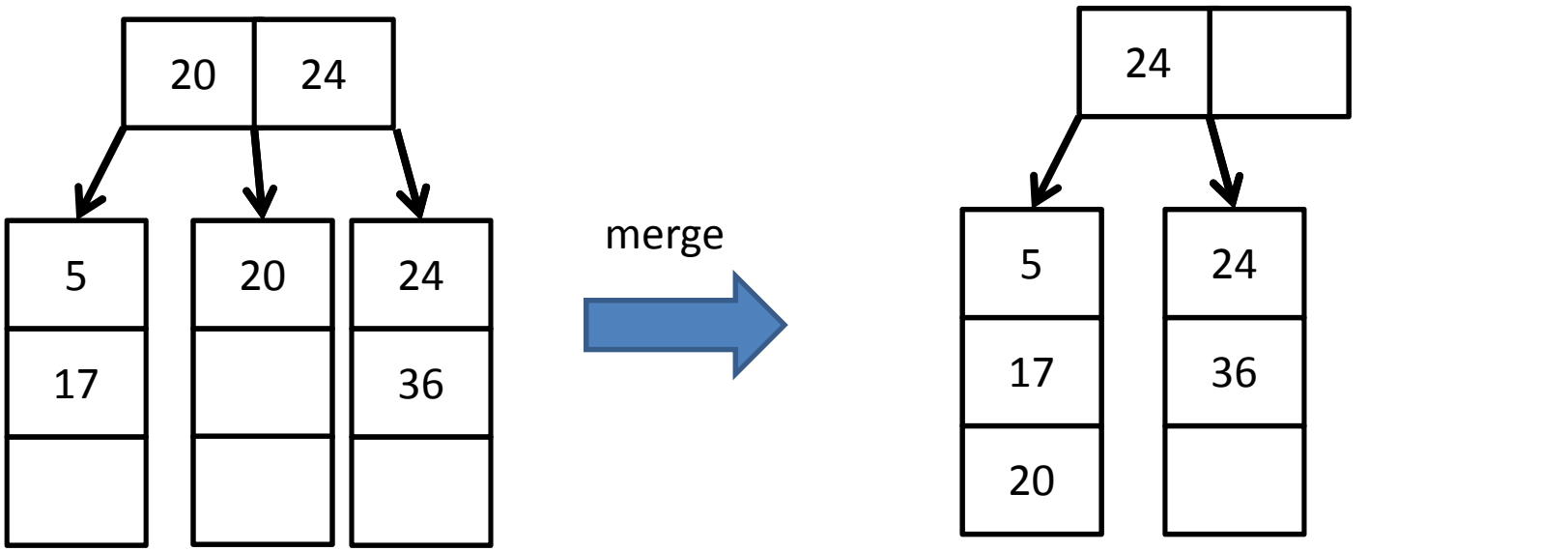
M=3, L=3



No underflow, so we're done.

Delete 22

M=3, L=3

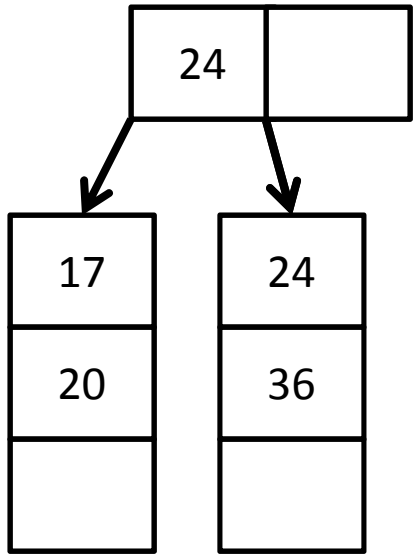


Deleting 22 causes underflow in middle child. We could adopt one of neighbor's children to fix underflow, except then, neighbor would underflow, so we need to merge.

After merging, fix the keys in the internal node.

Delete 5

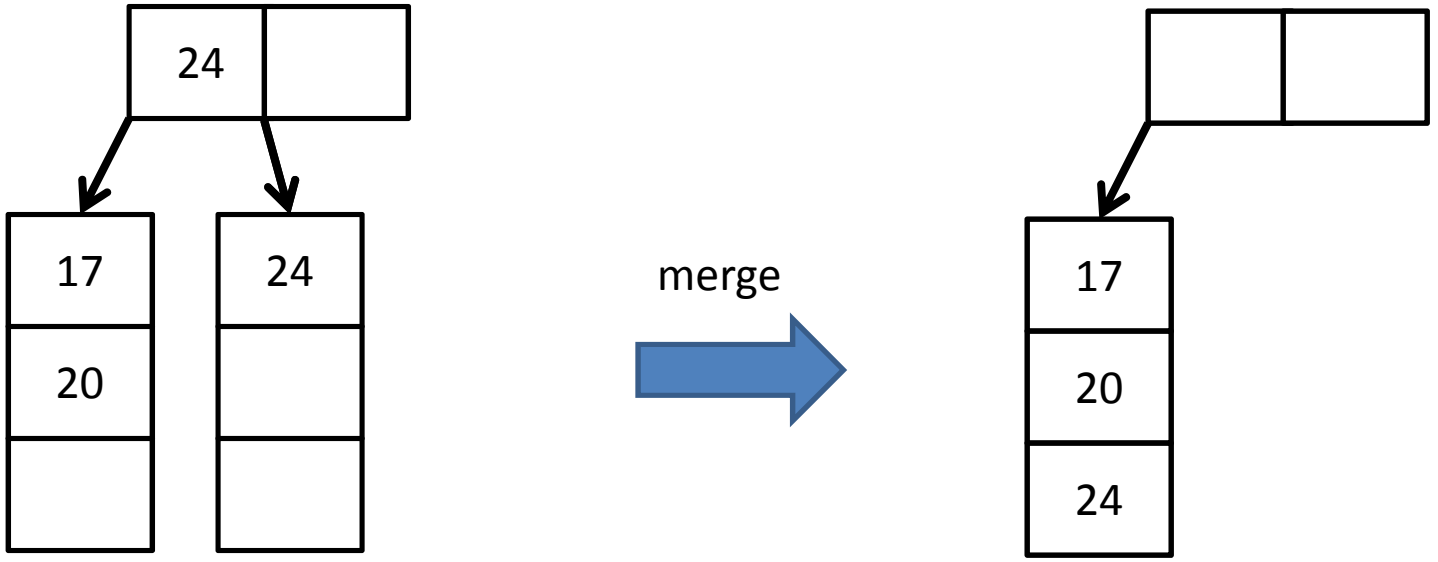
M=3, L=3



Deleting 5 doesn't cause underflow, so we're fine.

Delete 36 (step 1)

M=3, L=3

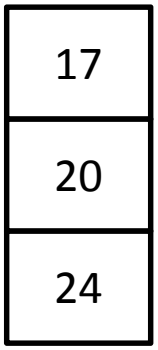


Deleting 36 causes an underflow, i.e. leaf now has ceiling(L/2)-1 items. Adopting from neighbor (17,20) would cause underflow in the neighbor, so need to merge.

Now we have an underflow of only 1 pointer in the parent node, so we need to fix it…

Delete 36 (step 2)

M=3, L=3



…so we delete it, and we're done.