Priority Queues

(Binomial Queues)
Chapter 6 in Weiss

CSE 326
Data Structures
Ruth Anderson

Today's Outline

- **Announcements**
  - **Written HW #2 due NOW**
  - **Project 2A due next Friday, 1/29**
  - **Written HW #3 due next Monday, 2/1**
- **Today's Topics:**
  - **Priority Queues**
    - **Skew Heaps**
    - **Binomial Queues**

Yet Another Data Structure: Binomial Queues

- Structural property
  - Forest of binomial trees with at most one tree of any height

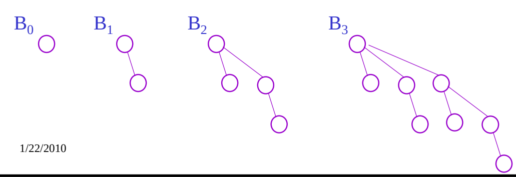
What's a forest?

What's a binomial tree?

- Order property
  - Each binomial tree has the heap-order property

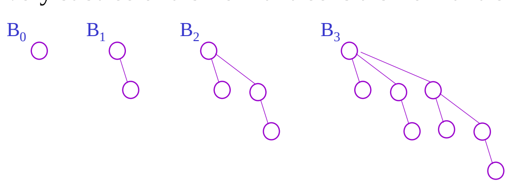
The Binomial Tree, B_h

- Height h
- Exactly 2^h nodes
- B_h is formed by making B_{h-1} a child of another B_{h-1}
- Root has exactly h children



The Binomial Tree, B_h

- Number of nodes at depth d is binomial coeff. $\binom{h}{d}$
  - Hence the name; we will *not* use this last property
- Every subtree of a binomial tree is a binomial tree



Binomial Queues

- Structural property
  - Forest of binomial trees
  - At most one tree of any height
- Order property
  - Each binomial tree has the heap-order property

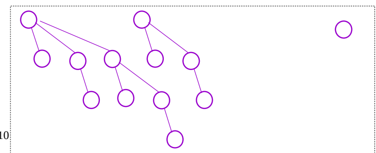
Binomial Queue with *n* elements

Binomial Q with *n* elements has a *unique* structural representation in terms of binomial trees!

Every binomial Q with *n* elements has this structure

Write *n* in binary: $n = 1101_{\text{(base 2)}} = 13_{\text{(base 10)}}$

1 B_3 1 B_2 No B_1 1 B_0



Properties of Binomial Queue

- At most one binomial tree of any height
- *n* nodes ⇒ binary representation is of size ?
  ⇒ deepest tree has height ?
  ⇒ number of trees is ?

Define: height(forest F) = $\max_{\text{tree T in F}}$ { height(T) }

Binomial Q with *n* nodes has height Θ(log *n*)

Operations on Binomial Queue

- Will again define *merge* as the base operation
  - insert, deleteMin, buildBinomialQ will use merge
- Can we do increaseKey efficiently? decreaseKey?
- What about findMin?

Merging Two Binomial Queues

Essentially like adding two binary numbers!

1. Combine the two forests
2. For *k* from 1 to maxheight {

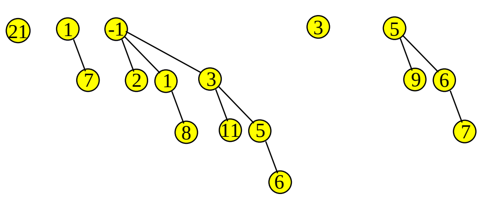
	# of 1's
a. *m* ← total number of B_k's in the two BQs	
b. if m=0: continue;	0+0 = 0
c. if *m*=1: continue;	1+0 = 1
d. if *m*=2: combine the two B_k's to form a B_{k+1}	1+1 = 0+c
e. if *m*=3: retain one B_k and combine the other two to form a B_{k+1}	1+1+c = 1+c

}

Claim: When this process ends, the forest has at most one tree of any height

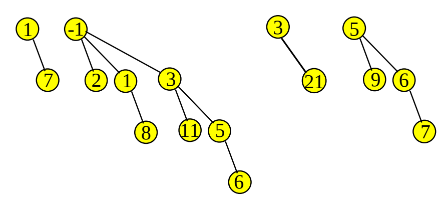
Example: Binomial Queue Merge

H1: H2:



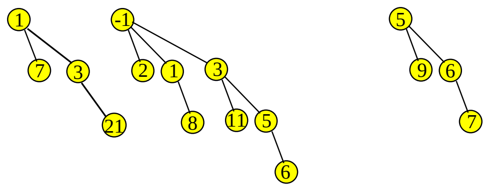
Example: Binomial Queue Merge

H1: H2:



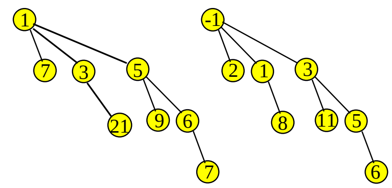
Example: Binomial Queue Merge

H1: H2:



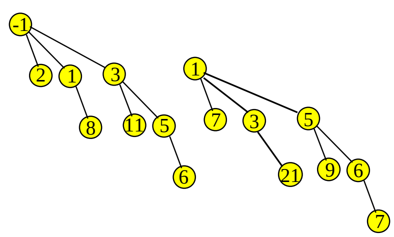
Example: Binomial Queue Merge

H1: H2:



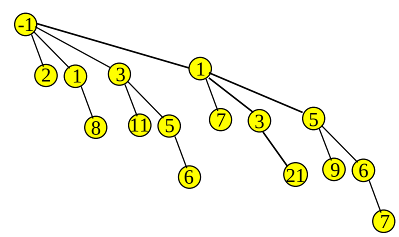
Example: Binomial Queue Merge

H1: H2:



Example: Binomial Queue Merge

H1: H2:



Complexity of Merge

Constant time for each tree

Max height is:

Number of trees is:

$\Rightarrow$ worst case running time = $\Theta($ $)$

Insert in a Binomial Queue

Insert(*x*): Similar to leftist or skew heap

runtime

Worst case complexity: same as merge

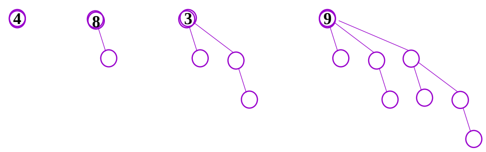
O( )

Average case complexity: O(1)

Why?? *Hint: Think of adding 1 to 1101*

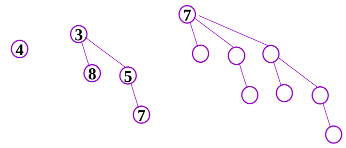
deleteMin in Binomial Queue

Similar to leftist and skew heaps….

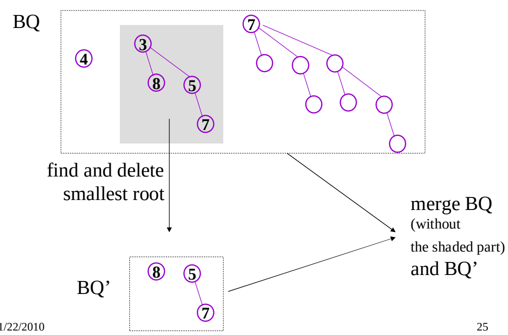


deleteMin: Example

find and delete smallest root

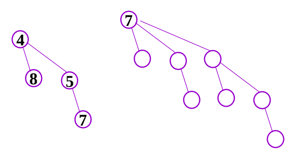


deleteMin: Example



deleteMin: Example

Result:



runtime: