Hash Tables

Chapter 5 in Weiss

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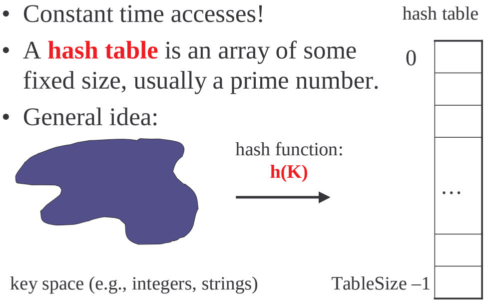
Hash Tables

- Constant time accesses!
- A **hash table** is an array of some fixed size, usually a prime number.
- General idea:



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Example

- key space = integers
- TableSize = 10
- **h**(K) = K mod 10
- **Insert**: 7, 18, 41, 94

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

3

Another Example

- key space = integers
- TableSize = 6
- **h**(K) = K mod 6
- **Insert**: 7, 18, 41, 34

0	
1	
2	
3	
4	
5	

4

Hash Functions

1. **simple/fast** to compute,
2. Avoid **collisions**
3. have keys distributed **evenly** among cells.

Perfect Hash function:

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Sample Hash Functions:

- key space = strings
- $s = s_0\ s_1\ s_2 \ldots s_{k-1}$

1. $h(s) = s_0$ mod TableSize
2. $h(s) = \left(\sum_{i=0}^{k-1} s_i \right)$ mod TableSize
3. $h(s) = \left(\sum_{i=0}^{k-1} s_i \cdot 37^i \right)$ mod TableSize

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Designing a Hash Function for web URLs

$s = s_0\ s_1\ s_2 \ldots s_{k-1}$

Issues to take into account:

h(s) =

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Collision Resolution

Collision: when two keys map to the same location in the hash table.

Two ways to resolve collisions:

1. Separate Chaining
2. Open Addressing (linear probing, quadratic probing, double hashing)

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Separate Chaining

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Insert:
10
22
107
12
42

- **Separate chaining**: All keys that map to the same hash value are kept in a list (or "bucket").

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Analysis of find

- Defn: The load factor, λ, of a hash table is the ratio: $\frac{N}{M}$ ← no. of elements ← table size

For separate chaining, λ = average # of elements in a bucket

- unsuccessful:
  λ
- successful:
  $1 + \lambda/2$

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How big should the hash table be?

- For Separate Chaining:

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tableSize: Why Prime?

- Suppose
  - data stored in hash table: 7160, 493, 60, 55, 321, 900, 810
  - tableSize = 10
    data hashes to 0, 3, 0, 5, 1, 0, 0
  - tableSize = 11
    data hashes to 10, 9, 5, 0, 2, 9, 7

Real-life data tends to have a pattern

Being a multiple of 11 is usually *not* the pattern

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Open Addressing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Insert:
38
19
8
109
10

- **Linear Probing**: after checking spot h(k), try spot h(k)+1, if that is full, try h(k)+2, then h(k)+3, etc.

Terminology Alert!

	“**Open** Hashing”	“Closed Hashing”
	equals	equals
Weiss	“Separate Chaining”	“**Open** Addressing”

Linear Probing

$f(i) = i$

- Probe sequence:

  0^{th} probe = h(k) mod TableSize

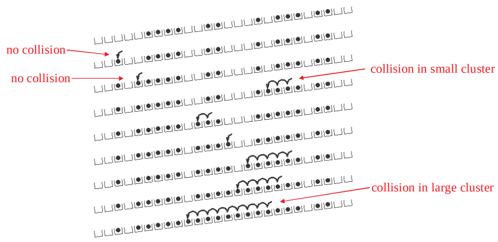
  1^{th} probe = (h(k) + 1) mod TableSize

  2^{th} probe = (h(k) + 2) mod TableSize

  . . .

  i^{th} probe = (h(k) + i) mod TableSize

Linear Probing – Clustering



[R. Sedgewick]

Load Factor in Linear Probing

- For *any* λ < 1, linear probing *will* find an empty slot
- Expected # of probes (for large table sizes)
  - successful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)}\right)$
  - unsuccessful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$
- Linear probing suffers from ***primary clustering***
- Performance quickly degrades for λ > 1/2

Quantitatively