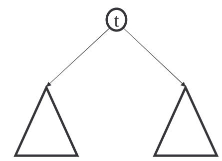
Trees
(Binary Search Trees)

Chapter 4 in Weiss

Tree Calculations

Recall: height is max number of edges from root to a leaf

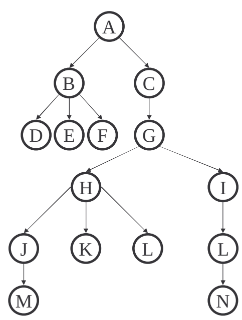
Find the height of the tree...



runtime:

Tree Calculations Example

How high is this tree?

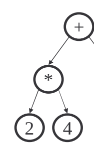


More Recursive Tree Calculations: Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

Three types:

- Pre-order: Root, left subtree, right subtree
- In-order: Left subtree, root, right subtree
- Post-order: Left subtree, right subtree, root



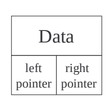
(an expression tree)

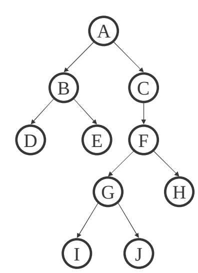
Traversals

```
void traverse(BNode t){
  if (t != NULL)
    traverse (t.left);
    print t.element;
    traverse (t.right);
  }
}
```

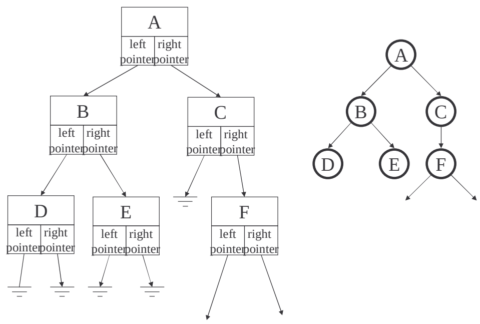
Binary Trees

- Binary tree is
  - a root
  - left subtree *(maybe empty)*
  - right subtree *(maybe empty)*
- Representation:

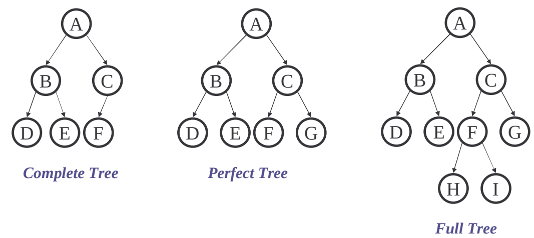




Binary Tree: Representation



Binary Tree: Special Cases



Binary Tree: Some Numbers!

For binary tree of height *h*:

- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

ADTs Seen So Far

- Stack
  - Push
  - Pop
- Queue
  - Enqueue
  - Dequeue
- Priority Queue
  - Insert
  - DeleteMin

Remember decreaseKey?

The Dictionary ADT

- Data:
  - a set of (key, value) pairs
- Operations:
  - Insert (key, value)
  - Find (key)
  - Remove (key)

insert(snyder, ….)

find(ppham)

- ppham
  Paul Pham, …

- snyder
  Larry Snyder
  OH: W 4:30-5:30
  CSE 584
- ppham
  Paul Pham
  OH: Th 2:30-3:30
  CSE 002
- Brianngo
  Brian Ngo
  OH: Tu 2:30
  CSE 002

*The Dictionary ADT is sometimes called the "**Map ADT**"*

A Modest Few Uses

- Sets
- Dictionaries
- Networks : Router tables
- Operating systems : Page tables
- Compilers : Symbol tables

Probably the most widely used ADT!

Implementations

	insert	find	delete
• Unsorted Linked-list			
• Unsorted array			
• Sorted array			

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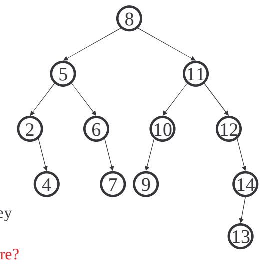
Binary Search Tree Data Structure

- Structural property
  - each node has ≤ 2 children
  - result:
    - storage is small
    - operations are simple
    - average depth is small
- Order property
  - all keys in left subtree smaller than root’s key
  - all keys in right subtree larger than root’s key
  - result: easy to find any given key
- What must I know about what I store?



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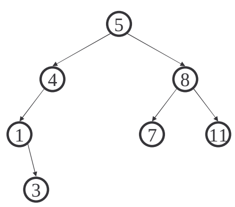
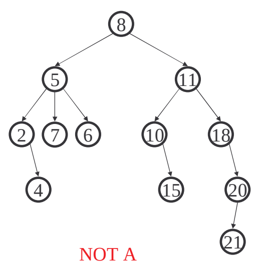
Example and Counter-Example



BINARY SEARCH TREE



NOT A
BINARY SEARCH TREE

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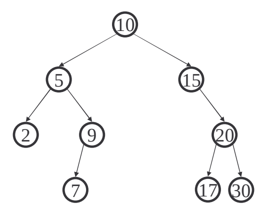
Find in BST, Recursive




```
Node Find(Object key,
          Node root) {
  if (root == NULL)
    return NULL;

  if (key < root.key)
    return Find(key,
                root.left);
  else if (key > root.key)
    return Find(key,
                root.right);
  else
    return root;
}
```

Runtime:

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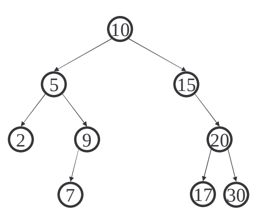
Find in BST, Iterative

```
Node Find(Object key,
          Node root) {

  while (root != NULL &&
         root.key != key) {
    if (key < root.key)
      root = root.left;
    else
      root = root.right;
  }

  return root;
}
```



Runtime:

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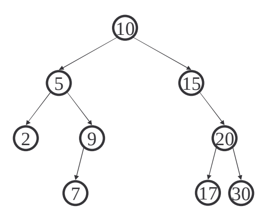
Insert in BST



Insert(13)
Insert(8)
Insert(31)

Insertions happen only at the leaves – easy!

Runtime:

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BuildTree for BST

- Suppose keys 1, 2, 3, 4, 5, 6, 7, 8, 9 are inserted into an initially empty BST.

  Runtime depends on the order!

  – in given order

  – in reverse order

  – median first, then left median, right median, etc.

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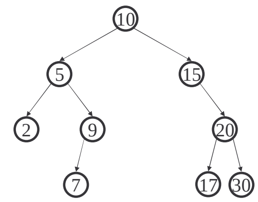
Bonus: FindMin/FindMax

- Find minimum

- Find maximum



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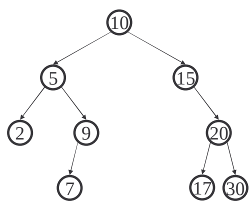
Deletion in BST



Why might deletion be harder than insertion?

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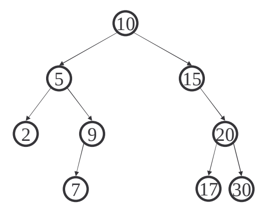
Lazy Deletion

Instead of physically deleting nodes, just mark them as deleted

+ simpler
+ physical deletions done in batches
+ some adds just flip deleted flag

– extra memory for deleted flag
– many lazy deletions slow finds
– some operations may have to be modified (e.g., min and max)



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Non-lazy Deletion

- Removing an item disrupts the tree structure.
- Basic idea: find the node that is to be removed. Then “fix” the tree so that it is still a binary search tree.
- Three cases:
  – node has no children (leaf node)
  – node has one child
  – node has two children

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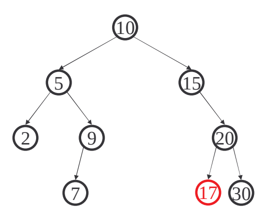
Non-lazy Deletion – The Leaf Case

Delete(17)



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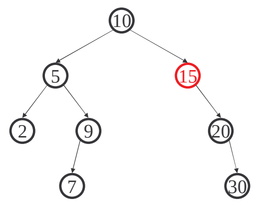
Deletion – The One Child Case

Delete(15)



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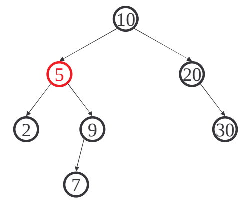
Deletion – The Two Child Case

Delete(5)



What can we replace 5 with?

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Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees!

Options:

- *succ* from right subtree: findMin(t.right)
- *pred* from left subtree : findMax(t.left)

Now delete the original node containing *succ* or *pred*

- Leaf or one child case – easy!

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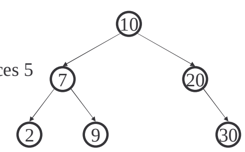
Finally…

7 replaces 5



Original node containing
7 gets deleted

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