$\{ a^i b^j c^k \mid i = j \text{ or } i = k \}$

$\{ a^n b^n c^n \mid n \geq 0 \}$

$\{ ww^R \mid w \in \{a,b\}^* \}$

$\{ ww \mid w \in \{a,b\}^* \}$

$\forall$ CFL A $\exists p$ s.t. $\forall s \in A$

if $|s| \geq p$ then $\exists u,v,x,y,z \in \Sigma^*$

s.t.

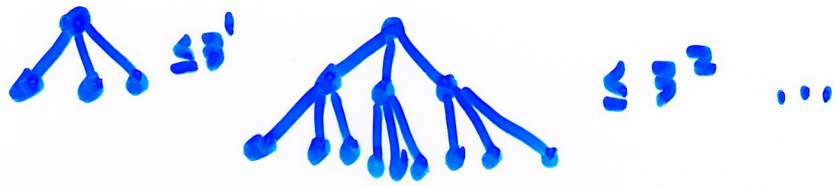
(i) $s = uvxyz$

(i') $\forall i \geq 0$ $uv^i x y^i z \in A$

(ii) $|vy| > 0$

(iii) $|vxy| \leq p$

Lemma: a b-ary tree of height h has $\leq b^h$ leaves



Conversely, more than b^h leaves implies height $> h$

Proof idea

G : a cfg for A

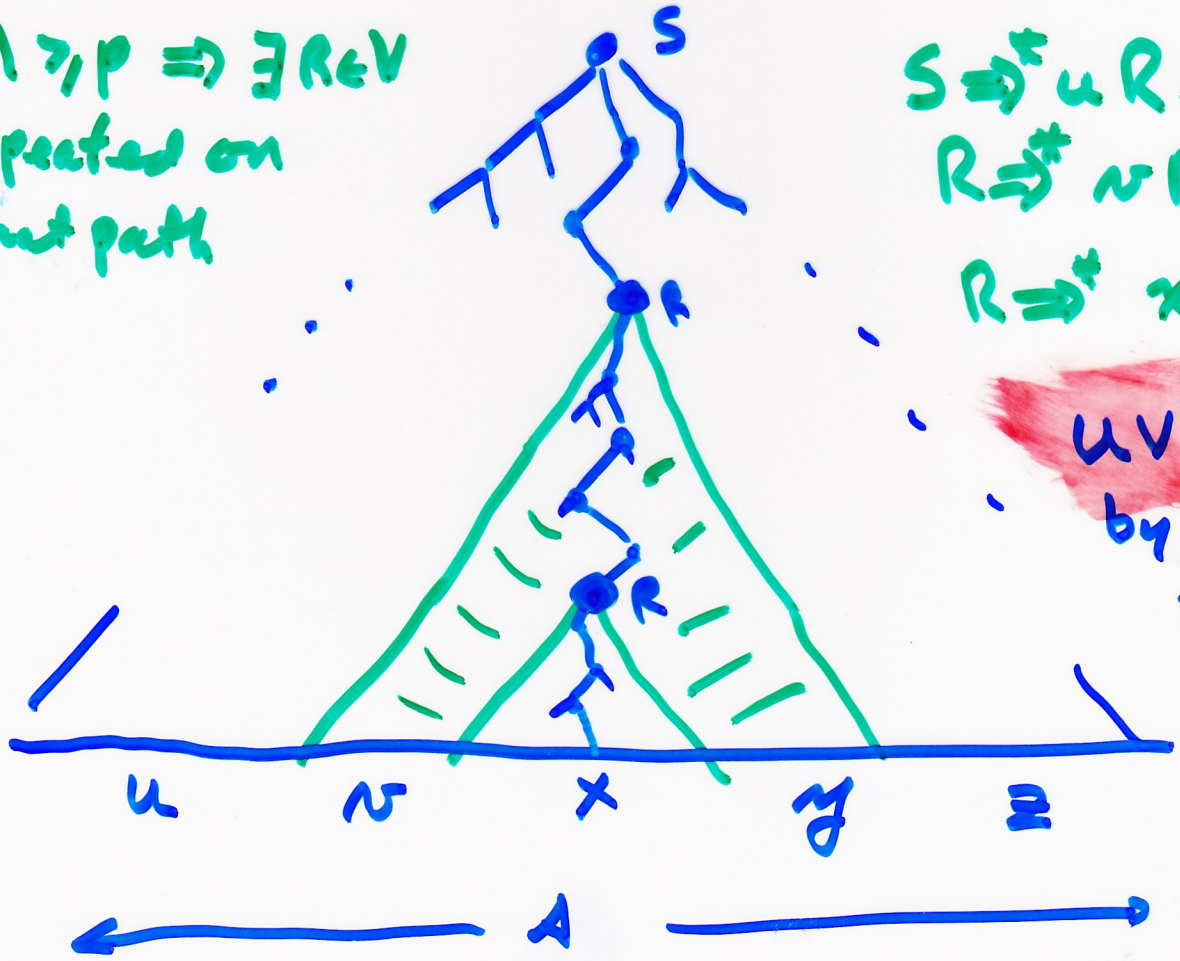
b = length of longest r.h.s of a rule in G

$p = b^{n+1}$ where $n = |V|$, # of vars in G

$s \in L(G)$ with $|s| \geq p$

Pick a smallest parse tree for s and a longest path in that tree

$|s| \geq p \Rightarrow \exists R \in V$ repeated on that path



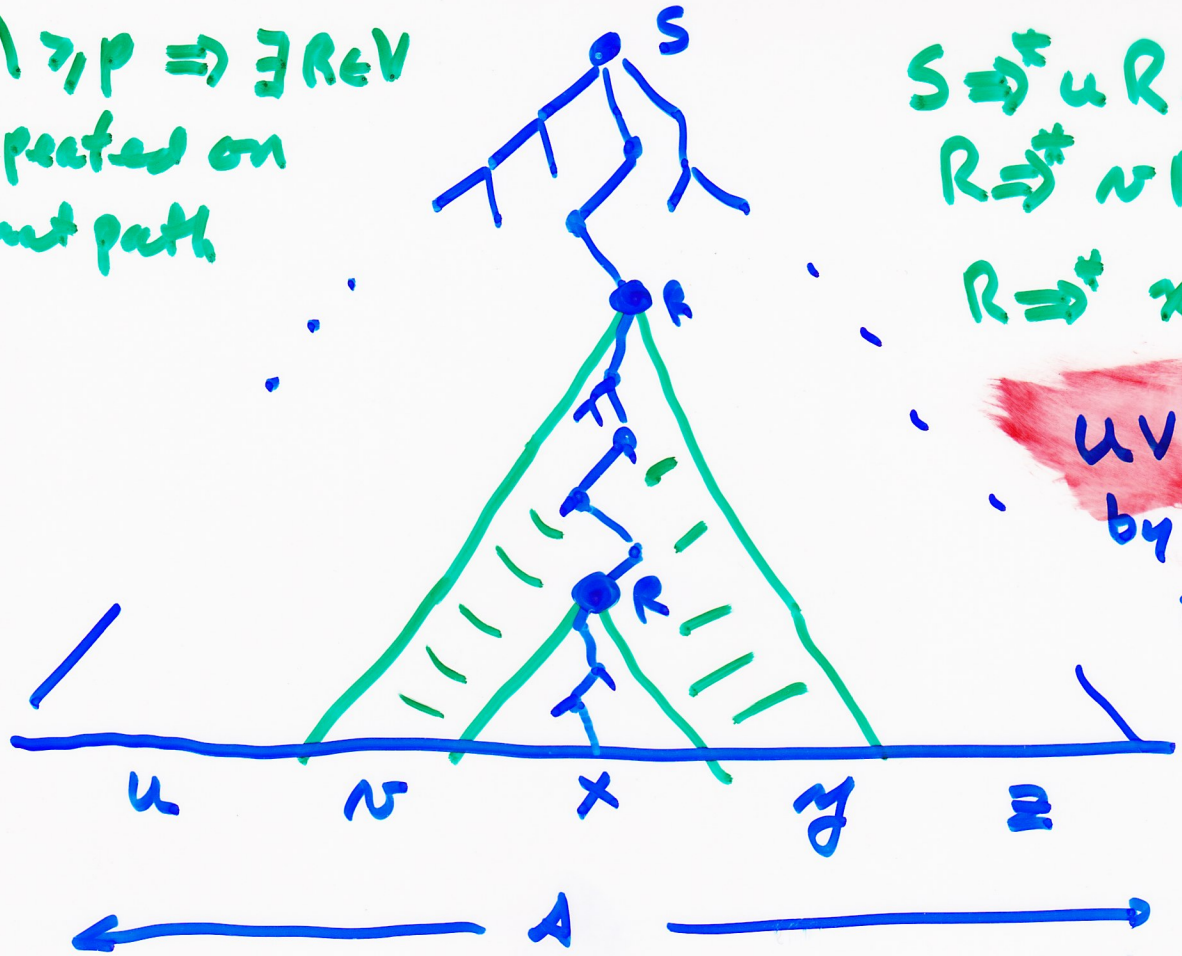
$S \Rightarrow^* uRz$

$R \Rightarrow^* vRy$

$R \Rightarrow^* x$

uv^ixy^iz by repeating i times.

$|w| \geq p \Rightarrow \exists R \in V$ repeated on that path



$S \Rightarrow^* uRz$

$R \Rightarrow^* vRy$

$R \Rightarrow^* x$

$uv^i xy^i z$ by repeating i times.

Why a repeat?

$> b^{|V|}$ leaves $\Rightarrow$ $> |V|$ path length

$\Rightarrow$ some variable R repeated.

Why $vy \neq \varepsilon$?

because it was <u>smallest</u> tree

Why $|vxy| \leq p$?

Pick repeat nearest leaf

$L = \{ a^n b^n c^n \mid n \geq 0 \}$

Let p be const from P.L.

Let $A = a^p b^p c^p$

By PL $\exists$ $uvxyz$ ...

vxy does not have both a & c

Since $|vxy| \leq p$

if no c in vxy then

uv^0xy^0z = $a^i b^j c^p$

for some $i \leq p, j \leq p$

$i + j < 2p$

Sim $|vy| > 0$

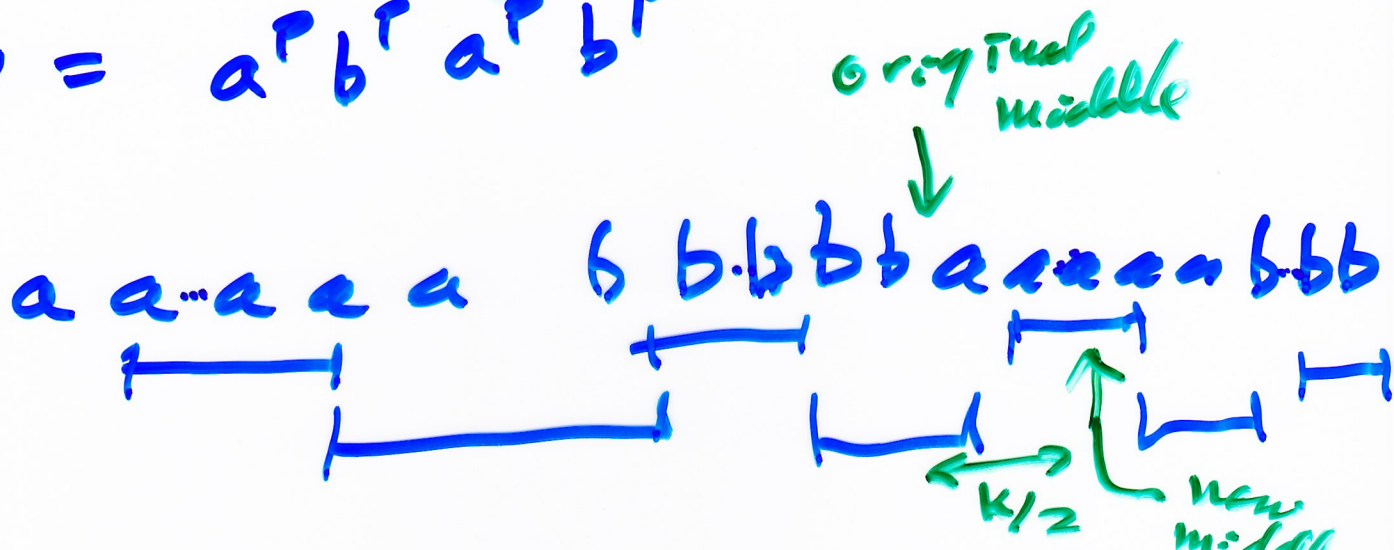
can't no a's

Contra

$L = \{ ww \mid w \in \{a,b\}^* \}$

~~$A = a^p b a^p b$~~

$A = a^p b^p a^p b^p$



$|vxy| \leq p$ ∴ confined to at most 2 adjacent blocks of a's & b's.

case 1 $|uvxy| \leq 2p$

$uv^0xy^0 =$ removed k letters from left half $1 \leq k \leq p$

Last letter of (new) left half is a, but last of right half is b.

∴ $\notin L$

Case 2

vxy in right half : similar

Case 3

vxy straddles middle.

$uv^0xy^0z = a^p b^i a^j b^p$

for some $i \neq p, j \neq p$

not both $i = j = p$

$i < j$ too few b's

$j < i$ ... a's

$i = j < p$ $a^p b^i \neq a^i b^p$

"Corollary"

$\{ww \mid w \in \{a,b\}^*\}$ not CFL "$\Rightarrow$" Java not CFL

This is representative Programming languages ~~that~~ that require variables to be declared (1st w) before use (2nd w), none of which (C, Java, C++, ...) are CFL's at this level.